THE ESSENTIALS OF LOGIC

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THE ESSENTIALS OF LOGIC

CHAPTER I

INTRODUCTORY

A Definition of Logic. Logic may be defined as *the science of the principles and conditions of correct thinking*. To the beginner, a definition serves mainly as a guide-post to direct his attention to the general character of the subject which he is about to study. The above definition should therefore be taken merely as an indication of the field. The terms of which it is composed will gather content and meaning as the various topics are covered. Toward the end of the course, it is to be hoped that the student will be able to come back to the definition with a fuller grasp of its significance.

Logic is a science in the sense that it is organized knowledge involving principles. The various sciences have different fields for investigation, but all of them agree in their purpose, which is the establishment of satisfactory information bound together and illuminated by laws. Thus, physics studies the most general characteristics of the physical world and seeks to reduce to order and interpret the facts it discovers; botany examines the structures, functions, and histories of plants; and psychology gives its attention to the behavior of creatures possessing consciousness. All these sciences seek to replace the loose and hazy notions of popular thought by exact and systematic knowledge. Now logic

has the same purpose, but its field is peculiar. It cannot be classed among the physical sciences which depend upon perception and measurement, nor among the biological sciences, nor, finally, among the social sciences. In a very real sense modern logic presupposes all these sciences and is somehow *a science at second remove from things*. It is a science about the *mental aspect* of the sciences. The mediæval logicians pointed out that the mind first 'intends,' or directs itself upon, the external world, and that only afterwards does it direct itself upon the mental processes and methods it has used. We may say, then, that logic involves a thinking about thinking.

Because man is naturally outward-looking and desirous of practical results he is seldom clearly conscious of his methods and of the mental processes involved in thinking. It requires an effort for him to take this new attitude and to think about thinking, about those mental operations which make knowledge possible. But when he does so, he finds that the mind does work in an orderly fashion. It is the nature of this orderliness that logic seeks to bring into clear consciousness.

The Value of Logic. So far as logic is a pure science, its value is essentially of the same kind as that of any other pure science. It satisfies an intellectual curiosity, a desire to know. Before a teacher can appeal strongly to such a value, the student must have experienced it; at some past time he must have forgotten himself as an individual with private ambitions in the enjoyment of seeing problems solved, of watching meaning replace confusion; he must have felt pleasure in knowledge for its own sake. The logician who is loyal

to his subject because he has faith in it asserts that logic has a value as a pure science, that man likes to know the working of his own mind just as he likes to know the laws of celestial mechanics. Why not, indeed? Is not the mind the supreme instrument which all must employ?

But a pure science need not lack practical value. *Logic is not only a pure science, but also an art or an applied science.* An applied science is one which is able to work out suggestions and rules based on a broad study of the field involved. Looked at from this angle, logic may be considered "a free study of some of the chief risks of error in reasoning." [1] What logic can do for an individual is to render him conscious of the best methods and the main difficulties in the various stages of actual thinking so that he will be more exacting in his mental operations. It should put him on his guard against dangers. It calls his attention incessantly to the value of clear and unambiguous ideas and to the proper level of scrutiny to be maintained before a fact is accepted, an inference drawn, or a proposition admitted as a premise in reasoning. The mind is an instrument, and it should be polished instead of being left in a state of nature. It is true that the exigencies of life force us to be critical, but such criticism is too rough-and-ready to be entirely satisfactory. Is there good reason to doubt that care of the mind will pay at least as much as care of the body? There are good mental habits just as there are good physical habits; and neither comes without effort and reflection. "Its [logic's] practical value in general education is firstly this: that it demands very

[1] Sidgwick, *Elementary Logic*, p. 8, Introduction.

careful and exact thinking about its own subject-matter, and thus tends to produce a habit of similar carefulness in the study of any other subject. In this it only does for the mind what a thorough training in any other science might do. Secondly, it makes us realize better what the general forms of speech that we habitually use really mean, and familiarizes us with the task of examining our reasonings and looking to see whether they are conclusive. In this it has an effect which the study of some special science like botany is not equally calculated to produce. Thirdly, it brings into clearer consciousness, as aforesaid, our ideal of what knowing is, and so furnishes us with a sort of negative standard; it makes us more alive to shortcomings in our ordinary opinions. But its chief value lies in its bearing upon those ultimate problems, concerning the nature of reality, and man's place and destiny in the world, from which at first sight it might seem far remote."[1]

If logic can do even a part of this, it justifies itself as a practical study. While logic does not create new capacities in the mind of an individual, it helps to train and sharpen the capacities already there.

The Kinds of Logic. Logic is one of the oldest of the sciences and has had a varied history. The result has been the growth of different branches emphasizing different aspects of thinking and somewhat different interpretations of the aim of logic. At times, these 'kinds of logic' have seemed opposed to one another, but, of late, they are seen to supplement one another and to make logic a broader investigation than it would have been without this branching. One kind of logic (exact

[1] Joseph, *An Introduction to Logic*, p. 10.

or symbolic) seeks to assimilate itself to mathematics in its methods; another (concrete) stresses its relation to general philosophy, especially to theory of knowledge; another (empirical or inductive) lays its emphasis upon the methods of the experimental and observational sciences. Back of these historically and linking itself with them in various ways — sometimes welcomed, sometimes an outcast — is what is called 'Formal Logic.' We shall have a better idea of the limitations of Formal Logic as we cover various topics such as the 'syllogism,' 'immediate inference,' and the 'logical form' of statements. Certain doctrines and technicalities must be discussed and given as true and adequate a setting as possible. They have their value, though this value must not be overestimated.

Introductory Logic. The Introductory Logic of to-day in America seeks to organize together the essentials of the various kinds of logic. It retains as much of Formal Logic as seems valuable and combines it with a concreter study of actual thinking with a view to determine the principles and conditions of correct thinking. In this effort, it has been aided immensely by the growth of psychology and philosophy, and the increasing reflection, by scientists themselves, upon their methods. Logic has acquired a new lease of life and is genuinely growing. More is known about thinking than ever before, and this added knowledge has stimulated logic to break away from formalities. Yet logic has a rich inheritance from the past which must not be belittled. What is needed is a new perspective and a more vital setting. We want to know the structure of our actual thinking so as to guard against the risks of error.

Relation of Logic to Other Sciences. The sciences most closely related to logic are rhetoric and psychology. The connection with psychology is obvious and we must examine it in some detail. But first let us examine its relation to grammar and rhetoric.

Thinking is intimately bound up with language; language is, in fact, a necessary instrument to any sustained thinking which involves general ideas. For grammar, language is itself the subject-matter; for logic, it is the instrument of thought. The function of words is the fixation of meanings in our own mind and their suggestion to others. In another chapter we shall discuss the relation between thought and language. At present we only wish to point out that *the law of language under its logical aspect is that of non-ambiguity.* Statements must, so far as possible, be clear as regards their significance. Accuracy and appropriateness in the use of words go hand in hand with the discrimination of ideas.

Rhetoric is more nearly akin to logic than is grammar. While grammar treats of words largely apart from their meanings, rhetoric concerns itself with the expression of thought. Such expression is studied in relation to the purpose in hand, which may be exposition, description, argumentation or persuasion. It is impossible to draw any hard-and-fast line between these disciplines, for each is influenced by the results of the other; we may say, however, that to-day both rhetoric and logic presuppose grammar as a preliminary and differ from each other in that which they stress. Logic is more interested in the *meaning* which is expressed, its implications, foundation, and validity, than in the

mode or style in which it is expressed. Rhetoric, on the other hand, culminates in the artistic use of language. The fact that language is a necessary instrument of thought brings these disciplines together; but logic has wider relations because thought is greater than its instrument. Language is, after all, a tool.

The logician reflects upon the mental processes by means of which knowledge is achieved and naturally finds much assistance in psychology. So far as he investigates *how* the individual thinks and the *conditions* of this thinking, he works hand in hand with the psychologist. Both examine perception, conception, judgment and reasoning and reach harmonious conclusions. The main difference between them is that psychology concerns itself more with the mental content, while logic stresses the value of the product. Just because the mind is a unity, it is impossible to find other than a working distinction between them due to a division of labor. They overlap, then, in the higher mental processes, and logic passes on to a study of the structure of knowledge and its validity, while psychology concerns itself with the structure of consciousness, its relation to conduct, and its physiological conditions and concomitants. The logician has the right to make use of the knowledge psychology has achieved which bears upon his own task. Their relation should be one of mutual helpfulness.

The Material of Logic. All sciences are objective; that is, they consist of recognized facts and of principles which organize and interpret these facts. Now, logic is, as we have already pointed out, a science about the sciences. *It is interested in the nature of proof and*

the laws of evidence; it desires to see how knowledge is built up and how the parts of knowledge depend upon one another. This interest cannot be satisfied by introspection alone, but requires a close study of the various sciences. In these it finds knowledge, and in their history the record of the steps by which it was obtained. We may say, then, that the material of logic is to be found in generally accepted knowledge and that this is approached with the purpose of discovering the processes involved. Knowledge is the material of logic much as the organism is the material of the biologist and the inorganic world the material of the physicist.

Inductive logic has always paid close attention to the sciences which have so splendidly developed since the Renaissance. It is an attempt to formulate the methods which grew up unconsciously, or largely unconsciously, in this new and wonderful extension of human knowledge. The assumption controlling the series of investigations which have given us the inductive branch of logic has been very well expressed by Whewell: "We may best hope to understand the nature and conditions of real knowledge by studying the nature and conditions of the most certain knowledge which we possess; and we are most likely to learn the best methods of discovering truth by examining how truths, now universally recognized, have really been discovered."[1]

But logic must concern itself with all sorts of thinking, with the conversations of daily life, with the arguments to be found in the social sciences, in political speeches, in essays, in editorials; otherwise, it is apt to

[1] Whewell, *History of Scientific Ideas* (3d ed.), vol. I, p. 4.

become too formal and pedantic and to lose touch with the flexibility and variety of actual thought. There is another reason why the logician puts this emphasis on less specialized thinking and refuses to let it be elbowed away by the more technical levels of thought. We can know what thinking is only by thinking ourselves and then retracing our steps in memory to see exactly what we have been doing. The material of logic can be found ultimately only in consciousness. And, if the subject-matter is to be realized and really reflected upon, it must be reproduced and revivified in our own thought. But it is not always easy to do this with past thinking when this is technical and has an atmosphere of its own. And, when all is said, familiar thinking illustrates the essentials of the thinking process. Any one who studies logic seriously must be willing to think for himself and to think about his thinking.

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CHAPTER II

THE NATURE AND SETTING OF THOUGHT

What is Thought? Thinking is the central operation for logic. It is the operation which underlies knowledge and opinion and makes them possible. It behooves us, therefore, to gain as clear an idea of its nature and conditions as we can. If it is affected by mental habits, we want to know what are the best habits; if it is controlled by feeling and emotion, we want to know how to lessen such control. In short, we wish to study the ways in which the human mind assimilates and develops knowledge — especially that knowledge which is called true.

It is not easy to define that mental process designated thinking. "No words are oftener on our lips than *thinking* and *thought.* So profuse and varied, indeed, is our use of these words that it is not easy to define just what we mean by them. Assistance may be had by considering some typical ways in which the terms are employed. In the first place, *thought* is used broadly, not to say loosely. Everything that comes to mind, that 'goes through our heads,' is called a thought. To think of a thing is just to be conscious of it in any way whatsoever. Second, the term is restricted by excluding whatever is directly presented; we think (or think of) only such things as we do not directly see, hear, smell, or taste. Then, third, the meaning is further limited to beliefs that rest upon

some kind of evidence or testimony. Of this third type, two kinds — or, rather, two degrees — must be discriminated. In some cases, a belief is accepted with slight or almost no attempt to state the grounds that support it. In other cases, the ground or basis for a belief is deliberately sought and its adequacy to support the belief examined. This process is called reflective thought."[1] Now, logic is chiefly interested in this higher level of thinking where ideas are under control and organized in relation to one another. In fact, many logicians define logic as the science of proof or evidence.

Certain terms are used by logicians as practically synonymous. Thus we shall have occasion to speak of *reflective thought*, *reasoning*, and *inference* at various times. These processes are much the same, though the context will show slight differences of emphasis. Taking them as essentially the same, our present task is to find out what is the nature of the mental operation involved. What do we do when we reason or infer? What we want now is a suggestion of the general character of the operation, leaving it to later chapters to study the various steps in methodical reasoning.

Discovery and Proof. There are two parts to any complete act of reasoning, discovery, and proof. We reason when we are confronted by problems, by unforeseen situations to which our habits and usual ideas are not immediately applicable. We are, as we say, at a loss; we don't know what to think or to do. But this perplexity lasts for a time only. We make shift to size up the situation, to compare it with what in the past seems most like it, to analyze the various features, and

[1] Dewey, *How We Think*, pp. 1–2.

so on, until some idea suggests itself as a possible solution. Such is the stage of tentative discovery. But another stage immediately sets in. The suggested solution must be tested or proved. In practical affairs, this proving consists, in the main, of overt action. The new idea is tried out to see whether it will work. The proud possessor of a new auto finds himself compelled to reason in this rough-and-ready way only too often. This practical level of reasoning is sometimes called 'trial-and-error.' The process is familiar to all.

In more theoretical reasoning, both the stage of discovery and the stage of proof are reflective. The scientist who makes a discovery has, as a rule, long brooded over the problem. The idea of universal gravitation did not come to Newton as a mere chance thought; his mind was ripe for it. Archimedes had long been pondering over his particular problem before he took his famous bath and discovered the principle of specific gravity. But after the brilliant idea has suggested itself, it must be *proved* in the light of the facts. It must be shown to *follow from* certain premises or data; it must be *guaranteed* by its reasons or grounds. And this second task is often the more perplexing of the two. By proof we mean the process of establishing an idea on a firm intellectual foundation. Newton had to work out the method of fluxions and make delicate calculations on the basis of known measurements before he could consider his hypothesis reasonably proved, while Darwin had to make numerous experiments and collect data of the most varied kinds before he was willing to publish his theory. Proof must be considered a very important part of reasoning. Scrupulousness in

regard to its standards is the sign of a trained mind. It is not too much to say that logic is primarily the science of proof. The question before the logician's mind is, 'Is such and such a given conclusion warranted?' Logic is retrospective and probative.

The Psychologist's View of Reasoning. In the preceding chapter, we came to the conclusion that logic and psychology overlap, that no hard-and-fast line can be drawn between them. It may be of interest, therefore, to examine the psychologist's view of reasoning. "To define reasoning fully it must be distinguished from imagination and memory when observed from the inside, and from instinct and habit when expressed in action. Reasoning may be distinguished from memory and imagination, not so much by the character of the mental states or by the way that they are obtained, as by the attitude that is taken toward them when they arise. The idea that is attained by reasoning may be exactly like an idea that on other occasions or by another man is merely remembered. The laws that govern the appearance of rational ideas are the laws of association, controlled in the same way as in memory and imagination. The important differences are: *that the results of reasoning are new and are accepted as true; the results of memory are true, but not new; and the results of imagination are new, but not true.*"[1] ? It is well to bear these contrasts in mind. Reasoning is a mental process taking some time to occur and controlled by the purpose of reaching accepted results. The logician makes the further demand that these results be established by relation to their ground.

[1] Pillsbury, *Essentials of Psychology*, p. 217.

Examples of Reasoning. Let us take a case of reasoning in regard to heavenly phenomena which was made quite early in the history of civilization. Man soon discovered that a shadow was caused by the intervention of an opaque body between a source of light and a distant surface. He did not know exactly what took place, but he knew that the light was cut off from the region back of the opaque body. Thus he arrived at the universal proposition, 'All shadows are caused by the intervention of an opaque body.' But an eclipse was soon recognized to be a case of shadow. Hence an opaque body must have intervened between the earth and the source of light. Here is a case of reasoning in which the conclusion is reached by resemblance or analogy. An eclipse is a case of the intervention of an opaque body *because it is a case of shadow*. This reasoning contains the ground. Let us now examine a case of bad reasoning. "According to some accounts, the wise men of Spain argued with Columbus that he could not reach India by sailing west, because if the earth were round, as he asserted, he would at some time reach a point where the ship would be going downhill and ultimately fall off, just as a minature vessel would fall, if it should attempt to travel around an artificial globe. If A is true (earth round), B must be true (circumnavigation impossible). If there is a resemblance between earth and artificial globe in contour of surface, there must be a further resemblance, so it was held, in the relation of each to the objects upon its surface. . . . In making their comparison, the wise men overlooked an important point of difference, viz., that for an object on the artificial globe the point

towards which it gravitates is outside the globe, while for objects on the earth this point is within the earth itself."[1]

Thinking may be more or less conscious and more or less methodical. Logic seeks to induce individuals to take thinking seriously, to make their thinking methodical and reflective. Especially does it stress evidence. Reflective thought is careful, and evidence-loving. It notes all the facts which can be thought to have bearing upon the problem and tests any conclusion by its consequences.

The Conditions and Occasions of Thought. Reflective thought or reasoning rests upon and grows out of experience. It has its causes and conditions. Thought does not arise just for no reason at all, but has its definite conditions. These reasons for thinking can be classified as social and personal.

The stimuli which lead individuals to think usually come from the social environment. The pity is that this environment is so moderately stimulating. The more there is the spirit of intellectual and social adventure, the more alive will men be. When the environment favors tradition and mere repetition and imitation, comparatively little vital and creative thinking is done. Life takes on the aspect of routine and habit, and classification of a formal kind overshadows discovery. Actions are repeated in the customary grooves, and no attempt is made to better them. There are periods, as during certain parts of the Middle Ages, when this social attitude is largely dominant. At such times, beliefs are handed down by tradition and are not criti-

[1] Bode, *An Outline of Logic*, pp. 2-3.

cally examined. Such an atmosphere is not favorable to thinking. The individual is not taught to analyze and observe, to doubt and to form hypotheses. Thus, there is a *sociology of logic*, or, rather, logic has its social aspects which must not be lost sight of. Let me, by means of an example, contrast the lethargy of thought during the later Middle Ages with the eager spirit of science at its best.

It is said that Francesco Sizzi, a Florentine astronomer, argued against Galileo's discovery of a new planet in the following way: —

"There are seven windows in the head: two nostrils, two eyes, two ears, and a mouth; so in the heavens there are two favorable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of nature, such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven.

"Moreover, the satellites are invisible to the naked eye, and therefore can have no influence on the earth, and therefore would be useless, and therefore do not exist.

"Besides, the Jews and other ancient nations as well as modern Europeans have adopted the division of the week into seven days and have named them from the seven planets; now, if we increase the number of the planets this whole system falls to the ground." [1]

The present is, on the whole, a period of skepticism and inquiry. The individual is taught to think for himself, along certain lines at least, and not to accept ideas

[1] Lodge, *Pioneers of Science*, p. 106.

unless he has pretty assured evidence for them. This social atmosphere, encouraging to thought, is a very important *condition* of thinking. Logic has a perfect right to champion such conditions. It is not called upon to be a passive judge of consistency alone.

The personal conditions of thought are also important. The prime condition is the recognition of a problem. Back of this recognition lies the attitude which faces problems and even goes to meet them halfway. This attitude is partly temperamental, partly the consequence of training. On the practical side of life, so long as our habitual ways of acting are sufficient to meet the conditions which confront us, there is little reflective thought because there is little need for it. There is mental activity, of course, because we must constantly recognize and classify things and situations; but this classificatory interpretation is usually so immediate that the individual does not need to pause, or, if he does pause, it is only for a moment. Difficulties arise, however, frequently enough, and the course of action is stayed. New adjustments must be worked out, and these adjustments depend upon the proper location of the difficulty and the discovery of its nature and remedy. Thinking intervenes at such a time. It is the sign and consequence of uncertainty, doubt, inability to interpret. How successfully the individual meets the problem depends upon his training, capacity, and character.

The Setting of Thought. Thinking arises and takes its course in the mind of an individual endowed with a fund of past experiences. The problem which occasions thinking involves a conflict within experi-

ence and calls into play all that is in any way relevant. Familiar ideas, prejudices, habits, memories, all enter into the council-chamber and demand audience. Thinking is the individual's experience in process of internal adjustment.

We may say, then, that thinking is purposive and selective. It is an expression of the growth of the mind. But the mind is distorted by old beliefs and prejudices and is very frequently deflected from the truth. It is like a river whose banks throw the current from side to side and produce in it all sorts of treacherous eddies. The logician must do the work of an engineer. He must point out the dangers which confront the mind which desires to think truly.

Why we think. Our study of the conditions and setting of thought enables us to answer the question, 'Why do we think?' There is both a practical and a theoretical urgency behind thought. Life demands that problems be solved in order that we may act wisely. When our habits push us in different directions, we must work out some suggestion which satisfies our mind or we are distraught. Besides, we know from past experience how dangerous it is to trust to a chance idea. We want to have a fair degree of assurance of the consequences before we act. To think is to rehearse the course of events beforehand and to meet every emergency with its appropriate interpretation.

But psychologists inform us that it is the very nature of the mind to work out harmonious ideas. The things we see are the products of adjustments. However far down we go in our mental life, we find the endeavor to harmonize various elements with one another. "The

sensations that are received are added to or changed to make them correspond more exactly to what from our different earlier experiences we know they must be. One always corrects the shape of the table-top as it appears in perspective. The angles are right angles as we see them, although the image must have acute and obtuse angles. One always makes an allowance for distance in the size of an object that is seen. The same object is always given the same size no matter how far away it may be, while the image on the retina diminishes constantly as the distance increases."[1] Thus, processes of adjustment are characteristic of the mental life at its different levels. This same tendency expresses itself in thinking. Thinking arises naturally and inevitably when the adjustment cannot be made almost automatically.

We have stressed practical problems as an explanation of thought; but theoretical problems are, at the present day, as fruitful in thought. These two types of problems are intimately interwoven by the very nature of life, for there are few ideas which do not have a bearing upon conduct. Yet the interests involved are fairly distinguishable. The growth of science has assisted man to develop interests of an impersonal kind, and about these interests and the investigations connected with them arise theoretical problems. Facts must be interpreted by theories and these theories must be harmonized so far as possible. The trained human mind is not satisfied with disorder and contradiction; it seeks to bring harmony out of chaos and to organize its ideas into a harmonious whole. Conflict in

[1] Pillsbury, *Essentials of Psychology*, p. 157.

any realm is to such a mind a stimulus to thought. The process of thinking has back of it all the forces and habits which demand order. To say that man is rational is to say that he seeks adjustment among his ideas.

Processes preliminary to Reflective Thought. The processes preliminary to reflective thought are, roughly speaking, two: perception and conception. We must have percepts and the ideas formed from them before we can think. Together, they furnish the material within which thinking goes on. It must not be supposed that percepts and concepts (ideas) are given apart from mental activity, but it is true to say that the higher mental processes, such as judging, analyzing, drawing conclusions, and argumentation, occur within a field of experience already fairly stable and definite. We see the external world, recognize various familiar objects in it, know what to expect of them, and have a pretty generous fund of ideas when we are engaged in the solution of any particular problem. The individual's mind is constantly adding to its store of information and its working system of distinctions, classification, and expectations. It is correct, then, to say that thinking represents the growing point of the mind, and that it leaves a sediment, so to speak, of ideas and habits and points of view. We must remember, however, that any part of this accepted knowledge may be challenged by new problems and new experiences; hence we must think of the mind as a living organism rather than as a building in which one story is added to another.

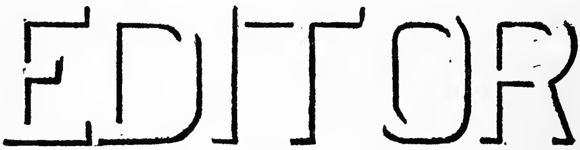
The psychologist has much to tell us of sensations, of

their qualities and intensities and number. This information does not have much bearing upon the subject-matter of logic, interesting as it is in itself. We know that the sense organs must be stimulated before we can have these sensations with which, in the rough and unanalyzed, we are all of us familiar. The next level after sensation is perception. It is of advantage to the logician to realize that perception is a complex process and that the percepts which we have as a result may be partly false. Whenever there is possibility of error, the logician cannot be indifferent. When we come to a study of scientific method, we shall have considerable to say about errors in observation. A few words about the general character of percepts and perception may not be amiss at this point.

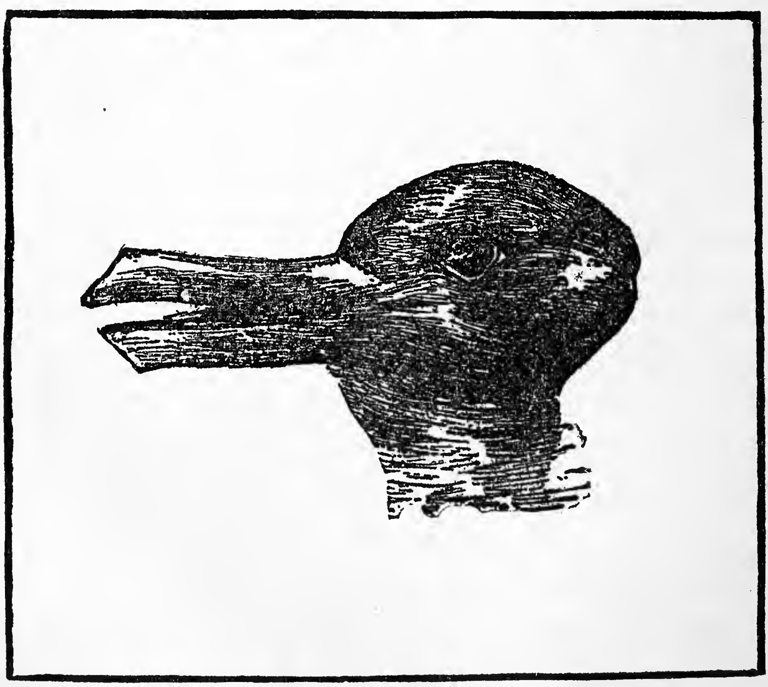
Put technically, percepts are the mental objects which result from the interpretation of sensory stimuli by centrally aroused processes. They are the primary mental objects of which we are aware when we use our senses. There is a sensory basis in all percepts; but there is, furthermore, a large element of applied past experience. A percept is really a mental product to the making of which there has gone much trial, selection, and adjustment. The cumulative result of these processes is given as an object, while the processes themselves seldom appear in consciousness. *So far as more is given in the percept than the senses warrant, the logician speaks of the inferential element in perception. It is because of this inferential element that perceptions may be erroneous.* We may see things which are not present because we take a certain sensory cue and add to it qualities which often have accompanied it

in our experience. What we shall unconsciously add depends largely upon the setting and upon the ideas which are occupying our minds. The following examples may make this possibility of error in perception more evident.

In the following figure taken from Jastrow, one can actually see dim lines suggested by the heavy lines that are drawn. These are supplied by memory.[1]



This second figure may be seen as a rabbit's head or as a duck's according to the thought which is uppermost.[2]



[1] Jastrow, *Fact and Fable in Psychology*, Fig. 2. [2] *Ibid.*, Fig. 19.

These illusions as they are called, serve to bring out the amount of interpretation which is at work in everyday life. Here is another example: "If a sportsman, while shooting woodcock in cover, sees a bird about the size and color of a woodcock get up and fly through the foliage, not having time to see more than that it is a bird of such a size and color, he immediately supplies by inference the other qualities of a woodcock, and is afterwards disgusted to find that he has shot a thrush. I have done so myself, and could hardly believe that the thrush was the bird I had fired at, so complete was my mental supplement to my visual perception." [1]

Speaking of such illusions, William James says: "Twenty times a day the lover, perambulating the streets with his preoccupied fancy, will think he perceives his idol's bonnet before him."

Conception arises upon perception as a foundation. Concrete, perceived objects are dissected into elements, and these elements, loosened from their particular context, are capable of repeated recall. Concepts are mental objects which are (1) not directly connected with the stimulation of sense-organs, and (2) are under our control so far as our having or not having them is concerned. The mental processes which make concepts possible are analysis and abstraction assisted by the method of comparison. We note the likenesses and differences between things. Certain qualities begin to stand out and are selected by the attention. These quali-

[1] Romanes, *Mental Evolution in Animals*, p. 324.

[2] The student will find James's discussions of illusions extremely interesting, especially pages 95 to 103 of the second volume of the *Principles of Psychology*. Münsterberg's *On the Witness Stand*, chap. I, is also suggestive.

ties and relations are then abstracted or considered by themselves, while the other features of the thing are disregarded. The result of this analysis is the isolation of the quality or relation from its context and our ability to recall it again and again at need. Such abstracted, repeatable mental objects are called 'concepts.' Other terms, sometimes employed, are 'meanings,' 'ideas,' 'universals.' This last term stands for a species of concept which is of special interest to the logician, and frequently used in philosophy. A universal is a concept which applies to an indefinite number of individual things. Any class name is a good example of a universal. We all have the *idea* of a horse, of an auto, of a soldier, etc. Such ideas are universals.

The ability to form concepts is essential to reasoning and reflective thought in general. Conceptual elements, obtained by comparison and abstraction, are organized together in all sorts of ways. Only by means of concepts are we able to rise above the particular field of concrete perception spread out before our senses. Knowledge is primarily an affair of conception, of rules and principles, facts and theories. But it must not be forgotten that these elements work back into and control our interpretation of the world we see about us. There is a constant and responsible interplay of perception and conception in our minds. Logic concerns itself with this interplay so far as it can be made the object of a critical reflection.

Fact and Theory. The distinction between percept and concept is more characteristic of psychology than of logic. For the latter, the more distinctive contrast is between fact and theory. In any argument, we con-

stantly seek to separate what may be considered the facts of the case from the theory which is added to it as an interpretation. Argument and dispute arise where fact and theory are not clearly separated; and the first step to the settlement of the argument is an agreement as to what is fact and what is theory. Now, fact is that which is admitted for the purpose of the argument; it is that which is, temporarily at least, considered indisputable. And it is interesting to note that fact in this sense may be either concrete or abstract. The Law of Gravitation is a fact because it is an assertion which I do not question. A theory, on the other hand, is that which is tentative and disputable. It needs support, proof, corroboration.

The first step in reflective thought or argument is, therefore, to distinguish fact from theory, the indisputable from the disputable. "In practice, assertions are seldom or never wholly untrue, especially when they correspond to a genuine belief. It is sometimes difficult, but seldom impossible (if it be thought worth while), to find a basis of agreement even with those whose view seems most opposed to our own. Some part of the way they go with us, and then the roads branch off. Why did they leave our road, or why did we leave theirs? That question and its answer is the beginning of argument. The matter of argument is always matter of opinion; not fact but theory; not fact but *inference* from fact."[1]

Knowledge a Growth. No problem or doubt which gives rise to reflective thought challenges all the individual's world. There is always fact as well as theory,

[1] Sidgwick, *The Process of Argument*, p. 18.

the accepted as well as the disputed. Knowledge is a growth and, like every growth, has a structure of its own. Logic has to pay considerable attention to the structure of knowledge and its medium, language. If argument is *knowledge in the making*, it would seem impossible to understand it apart from pretty adequate ideas of what *knowledge as made* consists in. All through logic, then, we see process and product, growth and achievement, dovetailed together. It is really impossible to understand one apart from the other. All problems arise within the knowledge already attained and reflect its structure. This fact explains why logic has to take up so many topics which seem linguistic or concern themselves with the divisions, classifications, and distinctions in knowledge rather than with discovery, argument, and reasoning.

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CHAPTER III

ABOUT TERMS

The Unit of Thought. The logician considers the judgment or the proposition as the unit of thought. The conclusion of a vital act of thought may find expression in a proposition, but in nothing short of it. Thus, in answer to a friend's question, I may say that a certain man is reliable; but I cannot use the words 'man' and 'reliable' in isolation and convey any assured meaning. The unit of thought is a complex idea, which is asserted or denied. Since thinking arises under the stimulus of a problem, its result must be definite, it must assert or deny something. This definite solution of a problem is called a 'judgment' and its verbal form a 'proposition.'

The proposition can be analyzed into elements which are called 'terms.' These terms exhibit the complexity of the proposition, and are always present in a *relation* characteristic of the concrete act of thought. 'Fire burns,' 'Justice is kingly,' are propositions in which there is both unity and complexity, or, better yet, unity in complexity. In trying to understand the judgment, we must carefully distinguish between the mental process of thinking which finds its goal in an accepted idea and the accepted idea itself. It is this latter which finds expression in the proposition and which is analyzed by the logician.

The Elements of the Proposition. A proposition is always analyzable into a subject and a predicate. The

subject is that about which something is stated, while the predicate is that which is stated about the subject. These two elements are called the 'subject term' and the 'predicate term' respectively. And this brings us to the definition of a 'logical term.' *It is any word or group of words which can serve as the subject or the predicate of a proposition.* But these two terms do not exhaust the proposition. There is, further, the word which stands for the relation between these terms. This is called the 'copula.' In the proposition, 'Abraham Lincoln was a wise statesman,' the subject term consists of two words and is a proper name, the predicate term contains three words, while the copula is a form of the verb 'to be.'

Words and Terms. It should be noted that the different parts of speech are not equally capable of acting as terms or as the nucleus of terms. Nouns, pronouns, verbs, and adjectives have the character or function which enables them to act as the essential part of a term or even as a term. Such words are called 'categorematic' or asserting words. Other words are not able to designate an object of thought, but are subsidiary to such designation. They are called 'syncategorematic.' Adverbs, conjunctions, and prepositions are connective or qualifying words, and are therefore classed as syncategorematic.

Since terms often consist of many words grouped together, categorematic words may be found combined together by means of syncategorematic words. In the proposition, '*Justice is kingly* though no *king be just*,' the words italicized are categorematic, while the others are syncategorematic. This distinction is, perhaps, more

grammatical than logical; but it serves to stress the fact that a logical term is other than a word.

Kinds of Terms. Terms fall into different classes according to the point of view that is dominant. The first division is into 'concrete' and 'abstract.' Concrete terms, again, are divided into singular or individual, general or class, and collective terms. Let us examine these divisions, which are the most important, before glancing at other kinds of terms.

An abstract term is the name of a quality or relation taken apart from its setting. In the study of conception, the nature of abstraction was referred to. To abstract is to draw off or take away. But an abstract term is not simply a word which calls up an idea which is conceptual in character, for all words do this, it is a word which signifies an attribute or relation taken apart from any individual object. It is the relation of the idea to realities which are thing-like which is disregarded. Thus, 'sweetness' is an abstract term because it signifies a conceptual object obtained by abstracting a quality of physical things and considering it by itself. The objects signified by abstract terms cannot possibly be perceived.

Some abstract terms are farther from sense-perception than are others. This is because the objects of which they are characteristics involve more construction. Thus, 'equality,' 'animality,' 'constitutionality,' 'discipline,' are abstract terms at second remove, so to speak. It is to be noted that abstract terms are always nouns.

A few words need to be said concerning the dangers involved in the use of abstract terms. Since they are nouns, they are readily thought of as referring to indi-

vidual things, more subtle than physical things, yet somehow real after the same fashion. The result is a fallacy which is usually called the 'fallacy of hypostatization.' Other writers speak of it as the 'fallacy of abstract terms.' It consists of treating abstractions as veritable things. "Our everyday use of such terms as 'conscience,' 'memory,' and 'will' shows further how ingrained is this habit of treating abstractions as though they were independent things. In reality these terms are the names of attributes. It is not uncommon, however, to find that conscience, for example, is conceived, in a vague fashion, as though it were some kind of thing, inhabiting the inmost recesses of the soul and performing the functions of an oracle. Similarly, there is no separate thing called memory or will, but only different instances of remembering and willing."[1]

In contrast to an abstract term, *a concrete term is a word or group of words which refers to a person or a thing, or a group of persons or things.* If there is any doubt whether a given term is abstract or concrete, the test to apply is, 'What does it lead us to think of?' If it lead us to think about definite individuals, it is concrete; if it lead us to think about attributes, it is abstract. Thus, 'man,' 'animal,' 'a society,' 'Lake Michigan,' 'a red apple' are concrete terms; 'humanity,' 'animality,' 'society,' 'redness' are abstract.

Concrete terms, we have said, are divisible into singular, general, and collective. These terms agree in their function of calling to mind *things;* they differ in the *number* of things implied and in the *way* they are taken.

A singular or individual term is one which applies

[1] Bode, *An Outline of Logic*, p. 49.

to only one thing. Such a term is selective and is used to identify, or direct our thoughts to a thing or experience which is regarded as a distinct existence. Examples: 'the large dictionary in this room,' 'the center of the earth,' 'the fifth page of this book.'

It should be noted that the point of view determines in large measure what shall be regarded as a single thing. A note of music may awaken an interest which leads to a selective emphasis on it so that it stands out from its setting, or it may be completely merged in the whole composition. It is worth while to reflect upon this *relativity* of what we shall consider a thing *to our interest.* We cut out things from continuous wholes much as the housewife cuts out cookies from the pastry. We divide up the world into larger or smaller parts in accordance with what attracts our attention. A mountain is a thing, so is a boulder on its summit, so is a peculiarly colored portion of the boulder, so is, perhaps, a molecule in it, etc. A singular term helps the individual to carry on this process of selective analysis, and enables him to direct the thoughts of another person to the same goal.

There are two methods for the formation of singular terms. The first method consists in the taking of a term which applies equally well to many things and adding new words until the application is obviously limited to one thing alone. The definite article always indicates the singular term. Very often the presence of the superlative form of an adjective or adverb helps to carry out this process of narrowing the application of the term to one thing. Examples: 'the greatest politician in the United States to-day,' 'the man who ran fastest in the meet.' The second method is to adopt a term arbitrarily

as a *sign* of a particular individual. Since language is social, a social element enters here. This use of the term must be consented to by the group. Personal names are of this character. Individuals are christened according to a socially recognized procedure. One is called 'Bernard Shaw'; another, 'William Smith'; another, 'If-Christ-had-not-died-you-would-have-been-damned Barebones.' It is interesting to note that a personal name is not always sufficient to select the one person meant. In that case, some descriptive word or words must be added. We all remember the 'Fiddler Smiths' and 'Little John Petersens' of our boyhood.

A general term is one which applies to any number of things. General terms are names of kinds or classes. Grammar makes the distinction between common and proper nouns which corresponds to a difference of numerical application. Examples of general terms are, 'dog,' 'member of Congress,' 'citizens of the United States,' 'holidays.'

The objects covered by a general term are members of a class. Such objects agree in certain characteristics and are classed together because of this agreement. These points of resemblance are called 'attributes' of the class. It should be noted that all objects have something in common, and that the number of possible classes is infinite. In both practical life and science, kinds or groups of objects having very much in common, are worked out and established as a matter of general knowledge. Such classes have accepted names. 'Salts,' 'halogens,' 'acids,' 'vertebrates,' 'mollusks,' 'constitutional monarchies,' 'fruits,' etc., are names of well-understood classes.

We shall have occasion to consider class names again when we come to the distinction between 'denotation' and 'connotation.' We shall see that general terms have two functions; they denote the individuals to which they *apply* and they connote the attributes which they *imply*.

Collective terms are a special class of concrete terms. They arise from the ability the human mind has of grouping objects together and considering them in their collective aspect. *A collective term is one which applies to a group of objects in which the individual members are lost sight of in the whole.* Such terms may be either singular or general. Examples of singular collectives are, 'the American navy,' 'the crowd before the courthouse,' 'the woman's club.' 'Navy,' 'family,' 'tribe,' 'state,' 'teacher's association,' are general collectives.

When we are referring to some specific group, the term is employed as a singular. 'The American navy is fairly efficient.' 'This crowd of students is out for a good time.' A general collective is used as a general term on the ground that there are various groups with the same characteristics. It is used *distributively* of the particular groups covered and *collectively* of the members of any one group. 'Excited mobs are dangerous.' 'Woman's clubs are becoming social centers.' This distinction will become clearer when we take up 'connotation' and 'denotation.' It is necessary to go beyond the mere form of the words to what is thought of in order to distinguish between singular and general collectives.

Ambiguity sometimes arises from the fact that the English word 'all' is used both distributively and col-

lectively. "That is, it may mean 'all taken together' or 'each and every.' Thus, we can say: 'All the angles of a triangle are less than two right angles,' and 'All the angles of a triangle are equal to two right angles.' In the former sentence, the word 'all' is used distributively, in the latter collectively. In Latin two different words are used: *cuncti* expresses the collective sense of 'all,' and *omnes* its distributive signification." [1]

Other Kinds of Terms. We have defined and examined abstract, concrete, singular, general, and collective terms. But terms are divided from other points of view into 'absolute' and 'relative,' and into 'positive,' 'negative,' and 'privative' terms. These distinctions are significant enough to justify some consideration.

An absolute term is one which refers to an object or quality considered by itself. The supposition is that such an object of thought has meaning in its own right; it does not depend on its relation to something else for an essential part of its meaning. Thus, 'stone,' 'building,' 'the State of Michigan' are absolute terms. *A relative term is one which derives an essential part of its meaning from its relation to something else.* Examples of relative terms are, 'shepherd,' 'patient,' 'ruler,' 'citizen,' 'pupil.' Relative terms usually go in pairs which imply one another and are then called 'correlatives.' Thus, 'teacher' implies 'pupils' and 'parent' implies 'child'; and the reverse is also the case.

It is sometimes said that all things are relative in the sense that they are interdependent. This is true, yet it does not invalidate the logical distinction we have just discussed. At the level of common sense, we usually

[1] Creighton, *An Introductory Logic*, p. 51.

ignore the relations which bind things to one another because they do not come out in perception. Only when relations stand out so clearly that they cannot be disregarded, do they form an essential part of our thought. Man is absolute, but a man *as a teacher* is relative. This term stresses a relation into which a man may enter.

But the term 'relative' is sometimes used in another sense. A word may involve a standard. 'Goodness,' 'badness,' 'luxury,' 'poverty,' 'beautiful,' and 'slowness' are examples of relatives of this kind. The danger lies in the fact that the standard is variable from person to person, from class to class in society, and from period to period in history. This variability, when not recognized, is a fruitful source of confusion. "To recognize and point out this character of relativity may on occasion serve to forestall much fruitless argument. For example, the question may be asked, 'Are the people of the present day more moral than those of the past?' Before we undertake to express an opinion we should ascertain what is meant by 'more moral.' To judge the past by present-day standards is one thing: to judge it by its own standards is quite another. Again, the question may be raised whether or not students who go into business are, as a rule, more capable than those who adopt the profession of teaching. Discussions of a question like this are prone to overlook the relativity of a phrase like 'more capable.' Until we specify whether we mean 'capable for business' or 'capable for teaching' or some other form of capableness, argument is likely to be futile." [1] The teacher in social ethics is

[1] Bode, *An Outline of Logic*, p. 46.

bound to meet questions which turn around the establishment of a *fair* standard for such things as expenditure. And this term 'fair' is itself obviously a relative term.

The distinction between positive and negative terms is obvious enough. *A positive term is one which implies the presence of a quality or qualities.* Thus, 'efficient,' 'selfish,' 'happy,' and 'heavy' are positive terms. *A negative term is one which implies the absence of such qualities.* 'Inefficiency,' 'disloyal,' 'unselfish' are negative terms. In the English language, such prefixes and suffixes as 'un,' 'in,' 'dis,' 'a,' and 'less' are used to form negative from positive terms. It should be noted that contrast-pairs formed in this way do not box the compass. Take, for example, the terms 'patient' and 'impatient'; a man may not be exactly patient while he is still far from impatient. The logician must call attention to this fact, however trivial it may seem, because words shift their meanings. When we come to study the possible manipulations of a proposition, such as obversion, we shall see why it is necessary to determine the exact opposite of a term. 'Intemperate' is not the contradictory of 'temperate.' A little reflection and some care in the use of words are all that is needed. Attention should be called to another point. Too much stress should not be laid upon the mere form of words. It is always best to penetrate to the meaning. Thus, such terms as 'invaluable' and 'unloosed' are positive in their meaning, while it seems nonsense to speak of 'intemperance' as negative. The absence of one quality means the presence of a condition just as positive. The negative form is an accident of language in such cases.

For the sake of completeness, we referred to the class of privative terms. *A privative term is one which signifies the absence of a quality which is usually present.* Because of the importance of this lack to which they refer, they are positive in form. Examples: 'maimed,' 'deaf,' 'blind,' 'orphaned.'

Connotation and Denotation. General or class terms have two functions: they apply to the members of the class and they serve to recall the common attributes of the class. Thus the term does double duty: it enables us to bear individual things in mind while thinking of them as belonging to a class, and it is a sign of the essential attributes of the class.

This double function of class terms reflects the way in which such terms develop. We notice that certain things have points of similarity, and, if these common qualities are important to us, we group these things together and think of them largely in terms of their common qualities. As a matter of fact, we never think of the things without some thought of the qualities, nor of the qualities without the implicit assumption that they are possessed by things. We may say, then, that denotation and connotation go together just as inevitably as do qualities and things. They are the reflection into the class term of this natural and constant distinction. It is for this reason that general terms denote things and connote qualities.

Synonyms for Connotation and Denotation. Instead of 'denotation,' some logicians speak of the 'extension' or 'scope' of the term. The corresponding names for 'connotation' are 'intension' and 'comprehension.' 'Denotation' and 'connotation' are prefer-

able because they have the verb forms, 'denote and 'connote,' and the adjective forms, 'denotative' and 'connotative.'

The Inverse Variation of Connotation and Denotation. The wider or higher the class, the fewer the attributes which the members of the class have in common. As you pass from a species to a genus in biology, the number of animals or plants covered by the term increases while the common attributes decrease in number. This double fact is summarized by saying that denotation and connotation vary inversely. This variation does not, however, follow any mathematical law. Increase the number of qualities possessed in common, that is, widen the connotation, and the members of the class possessing these qualities will be fewer in number. The following series will illustrate this principle: 'man,' 'Americans,' 'Americans of European descent,' 'native-born Americans,' 'native-born Americans who are citizens of Michigan,' etc.

It should be noted that, in our actual thinking, the connotation of a term is more apt to be explicit than the denotation. We do not actually run over the various members of the class, but, instead, have some individual vaguely in mind as a specimen.

Terms and Meaning. All words have meaning. This meaning varies from the most substantive and definite to the most fugitive and fluctuating. The meaning of a word is inseparable from the context in which it is used. It is for this reason that both the logician and the psychologist hold that the sentence or even the paragraph is the lowest unit of thought. A word or a group of words constitutes a term, and this term has a

complex meaning. It is, however, best to distinguish between the *meaning* of a term and *connotation.* In the first place, the distinction between denotation and connotation comes out clearly only for class terms; in the second place, the meaning of a class term includes both its denotation and its connotation. We mean the objects we think of as well as the qualities they have in common.

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CHAPTER IV

THE USE AND MISUSE OF LANGUAGE

The Necessity for Language. Logic deals with thinking and with knowledge which is the resultant of thinking. But thought is impossible without language. It is for this reason that logic has to concern itself so fully with language and its relation to thought. "Language is not merely an accompaniment of ideational activity; it is an instrument essential to its development. It is an appropriate means of fixing attention upon ideally represented objects as distinguished from percepts. It becomes the more necessary the more abstract ideal representation is, — in other words, the less it contains of the concrete details of actual sense-perception. . . . Within the mind of the individual thinker it serves to fix attention on the object of his own ideas; in communication with others, it serves to fix the attention of the hearer on the ideally represented objects present to the mind of the speaker."[1] As we advance in our study of logic, we shall realize, ever more clearly, what an intimate part language plays. Systematic knowledge is bound up with the use of a definite set of symbols, though these symbols may be of various kinds. Each special science works out its own technical terms, while mathematics has advanced step by step with its notation.

Language and Analysis. In a preceding chapter[2]

[1] Stout, *Manual of Psychology*, p. 462.

[2] Chapter II, "The Nature and Setting of Thought."

we suggested that reasoning is based upon the perception of likeness and difference. Now, language assists such discrimination by giving names to the common factors and aiding in their recall and free manipulation apart from perception. Gradually a world of ideas is built up in which man may freely move. Words are the *means* by which such ideas are fixated. It has been proved by experiment that we both observe and remember features of our experience better if we possess words to attach to them.

The Logical Law of Language. The logical law of language is non-ambiguity. Actually, words are indefinite and misty on their edges. We use them in a rough-and-ready fashion. We feel their appropriateness and develop habits in their use. The consequence is that there is not enough delicacy in discrimination; we tend to class things together when we notice that they have common features. Yet the differences may be of more significance for the problem in hand than the similarities.

The neglect of relevant distinctions, when these are important for clear thinking or adequate statement, is ambiguity. The ideal of the logician is to encourage individuals in those mental habits of accuracy and discrimination which are calculated to prevent ambiguity. The logical law of language is non-ambiguity, an *ideal* to which we should approach ever more nearly.

Causes of Ambiguity. There are many causes of ambiguity, but the basic cause is mental. Ideas are allowed to remain vague, and, as a consequence, they overlap and get into one another's way. Just as things are often classed roughly together because their differ-

ences are not noted, so ideas are only too frequently merged and confused. But clear and accurate thinking depends as much upon the awareness of differences as upon the awareness of similarities. Only he who is willing to take the trouble to distinguish things and ideas which are ordinarily grouped roughly together, can escape the pitfall of ambiguity.

But ambiguity has a social significance. Incorrect and inadequate thinking when expressed in language appears as ambiguous statements and arguments. Such statements are apt to deceive the unwary because they also refuse to make an analysis of the ideas involved. Hence ambiguities tend to propagate themselves and father a series of fallacies.

Now, while unwillingness or inability to analyze the various fields of experience, in order to build up clear-cut ideas not easily confused, is the basic cause of ambiguity, there are other causes at work, the variation of the meaning of words with the purpose or general context, and the imperfections of language as an instrument. It is difficult for language to keep pace with the subtlety and flexibility of thought. It is so easy to substitute the *average meaning* for the particular meaning which the situation demands. Again, a word may have, and usually does have, more than one meaning of an average sort; and, to make matters worse, words are constantly changing their meanings. We always have more ideas than words with which to express them. It may be said that this ability of a single word to do duty for more than one idea is a perfection rather than an imperfection, but it certainly brings its dangers. A sharp eye for the context or purpose is the main safe-

guard. If this is not obvious to the reader or listener, ambiguity is unavoidable.

Univocal Words. Words which have only one meaning are called 'univocal.' The best examples of univocal words are to be found in the technical terms of the special sciences. In these, a term is adopted, or made, as an accepted sign of a definite idea reached by painstaking investigation. Consequently, the danger of ambiguity is at a minimum. To secure such carefully delimited ideas is one of the chief purposes of any science; it is a recognized condition of advance. The technical terms of a special science when taken together form its nomenclature. Thus, 'molecule,' 'electron,' 'vertebrate,' 'cell,' 'angle' are technical terms. It should, however, be noted that not all technical terms are strictly univocal; it is only when they are given in the context of their special field that they have but one meaning. But this context is easily recognized.

Equivocal Words. Words which have more than one meaning are 'equivocal.' The vast majority of words which do not apply to material things are equivocal, as can be seen by glancing over the columns of any large dictionary. Thus, to take a simple word like 'stop,' we find that it means (1) 'to close,' (2) 'to confine,' (3) 'to arrest,' (4) 'to interrupt.' Besides these popular uses, it has technical meanings in music, rhetoric, optics, horticulture, and finance. It is no wonder that a foreigner has difficulty in learning another language, and I am sure that, if all the humorous stories told in this connection are not true, at least there is ground for most of them. The truth is that there are not enough words to go around, and we are

obliged to make one word do the duty of many. A perception of the setting is essential if ambiguity is to be avoided. The only practical advice the logician can offer is to seek for the particular meaning and not trust the average or general meaning of the term.

Words change their Meanings. There are several tendencies at work to change the meanings of words. The three most important are generalization, specialization, and transfer of meaning by association and analogy.

At first, a word is the sign of a concrete thing or class of things. It is later extended to all those things which have a conspicuous attribute in common. As time goes on, extensions to more abstract objects take place. This change of application from a concrete particular to analogous things and objects of thought is called 'generalization.' "The word 'law' has undergone expansion under the pressure of physical research. Regularity, uniformity, constancy of occurrence is a much wider idea than bare authoritative prescription enforcing obedience under the threat of punishment — which is the simple juridical conception."[1] How easily ambiguity arises when two meanings coexist can be seen from the following quotation: "The existence of a power is even implied in the phrase 'laws of nature,' constantly used by science; for wherever there is a law there must be a lawgiver, and the lawgiver must be presumed capable of suspending the operation of law."[2] Another instance of this ambiguity of the word law has come to light in recent discussions bearing upon the nature of 'international law.' The English word

[1] Davidson, *The Logic of Definition*, p. 5.

[2] Goldwin Smith, *Guesses at the Riddle of Existence*, p. 143.

'law' translates what is expressed in continental tongues by two words; in German by *recht* and *gesetz*, in French by *droit* and *loi*. Is there an international law? If we mean by 'law,' *gesetz* and *loi*, no; if we mean *recht* and *droit*, yes. It is obvious that we have too few words in such a case.

Other simpler examples of this principle of generalization are 'fluid,' 'acid,' 'curve,' and 'oval.' We pass from the particular to the more general and abstract.

Again, words may take on a more special meaning. 'Priest,' 'minister,' 'captain,' 'clerk' are examples of this tendency to select a special application of the more general meaning and to ignore the other meanings. One of the most interesting cases of this specialization which has led to ambiguity is the word 'prove.' Originally it meant 'to test' as in the saying, "The exception proves the rule." Now it means 'to test successfully.' I presume that many a one has wondered how the exception could *prove* the rule.

There is, finally, the tendency to transfer a word to analogous or associated objects. Thus, the word 'church' is used to designate the religious body, the building, and some temporary group of worshipers. We speak of the 'chair' when we mean the presiding officer, or of the 'bench' when we mean the judiciary. It would be right to list under this heading those epithets which are based on felt analogies. Such epithets startle the mind and force it to recognize similarities which would otherwise have passed unnoticed. It is the poet, with his keen sensing of likenesses and differences, who is most apt to bring far-removed aspects of experience together by an illuminating phrase. We feel the appropriateness

of the expression while we hesitate to analyze it, so subtle and elusive is the connection.

These classes of changes in the meanings of words make us realize what a complex instrument language is. It is necessary to know all the essential meanings of commonly used words and to have some idea of how new shades of significance are added.

Vagueness. Vagueness is often the cause of ambiguity, but it is not the same as ambiguity. Our ideas of the meanings of the words we use may be hazy, but, so long as the subject we are talking or thinking about is simple, this lack of definiteness in our ideas may not lead us astray. So long as the problem is general and relatively practical, no great measure of analysis is needed. We must remember that words are frequently used just to indicate things and to suggest plans of action and desirable attitudes. Thus, common nouns are seldom used incorrectly even though the majority of people would be hard put to it to give an exact definition. Yet, even here, there is doubt when those exceptional cases are met which fall on the dividing-line between two classes. Is this man a doctor or a quack? Is this lawyer a genuine lawyer or a pettifogger? The trouble is, of course, not so much one of language as of classification. Classes shade into one another and their edges are blurred. Accuracy and completeness of classification are the pre-conditions of clear ideas.

The reason for vagueness in the understanding and application of words can partly be traced to the method by means of which the individual's vocabulary is developed. Words are picked up in their special applications and then extended to what are judged to be similar

cases. "The mode in which words are learned and extended may be studied most simply in the nursery. A child, say, has learnt to say 'mambro' when it sees its nurse. The nurse works a hand-turned sewing machine, and sings to it as she works. In the street the child sees an organ-grinder singing as he turns his handle: it calls 'mambro'; the nurse catches the meaning and the child is overjoyed. The organ-grinder has a monkey: the child has an India-rubber monkey toy: it calls this also 'mambro.'"[1] The individual's vocabulary is built up under social control only of a rough-and-ready sort. It is adequate for general purposes, but not where accuracy is needed or where slight distinctions are very important. Where the demands are more exacting, vagueness is treacherous and leads to ambiguity.

Abstract Terms particularly subject to Vagueness. Where there is no chance to appeal to perception as a test for the meaning of a term, vagueness is almost certain to exist. We can pick up the word just as the child does, but it is another thing to reproduce the idea. That requires mental activity of a high degree. The logician stresses the fact that concepts are primary and words secondary. What is meant by such terms as 'happiness,' 'liberty,' 'social justice'? I who have spent years in analyzing their meanings feel that I have a right to be skeptical of some glib use of the terms. "When we come to words of which the logical concept is a complex relation, an obscure or intangible attribute, the defects of the popular conception and its tendencies to change and confusion are of the greatest practical importance. Take such words as 'Monarchy,' 'tyranny,'

[1] Minto, *Logic, Inductive and Deductive*, p. 83.

'civil freedom,' 'freedom of contract,' 'landlord,' 'gentleman,' 'prig,' 'culture,' 'education,' 'temperance,' 'generosity.' Not merely should we find it difficult to give an analytic definition of such words: we might be unable to do so, and yet flatter ourselves that we had a clear understanding of their meaning. But let two men begin to discuss any proposition in which any such word is involved and it will soon be found that they take the word in different senses."[1] Such terms are really technical and should be treated as such. The difficulty is, however, practical as well as scientific. People are obliged to think about such subjects because of their duties and needs as citizens, and because such terms are frequent causes of dispute. What the logician must warn against is dogmatism. The average man is not easily persuaded that familiarity with a word does not involve knowledge of what it means. Such dogmatism is one of the chief causes of error.

Logic and Language. Logic deals with thought, but we can get at thought only through language. Now, language is not a transparent medium; it veils as well as expresses the ideas and purposes which seek utterance and the understanding of other minds. The logician must, therefore, be equal to the emergency of interpretation if sufficient clues are given. "The practicing logician, ever seeking behind the accidental parlance the necessary sequence of idea, studies the idiomatic expression of thought, with which, however, and not with the expression, his concern truly lies."[2]

[1] Minto, *Logic, Inductive and Deductive*, p. 86.

[2] Macleane, *Reason, Thought, and Language*, p. 9.

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CHAPTER V

CLASSIFICATION AND DIVISION

Classification and Classes. In our study of terms we distinguished class, or general, terms from the other kinds of terms. It is our present purpose to study in more detail the nature of classes and the character of the processes by which they are obtained. We shall see that the establishment of classes involves a large measure of mental activity. Certain classes suggest themselves quite naturally, while others are formed only after a deal of effort has been expended in the discovery of hidden relationships between things apparently different in nature. At whatever level classes are formed, comparison, discrimination, analysis, and construction are at work, though with different intensities. It must never be forgotten that an object is not a member of a class in its own right, but because a class has been conceived by the human mind. Man and nature collaborate in the construction of classes. When a class has been established, however, its membership is potentially determined by that very fact. Having conceived of vertebrates as a class, we cannot refuse membership to this or that creature because of our dislike for its bad habits or its ugly appearance. The Gila monster is as good a vertebrate as the most docile pussy that ever slept in her mistress's lap.

We may define *classification as the process of grouping things together according to their possession of cer-*

tain selected common attributes. The things possessing these attributes are the members of the class, while the class is the members taken together. Such classes may be relatively permanent or quite temporary and occasional. The following quotation will illustrate the wide range of classes which the logician recognizes: "By a class will here be meant any imagined group of individual cases, whether real or unreal — a group in which every individual is supposed to resemble all the others in some respects, though differing in others. There are classes of actions and events just as of everything else; 'miracle' is a class name, for instance; or 'coronation,' 'battle,' 'eclipse'; in fact any name which is used so as to admit of a plural — either simply, as 'miracles,' 'negroes,' 'battles,' or in the more circuitous form of 'pieces of gold,' 'cases of deceit,' and so on."[1] There are filmy classes as well as the more ponderous classes of science. Both have their place for the mind of the logician.

The Need for Classification. The world man is confronted with is very complex. It consists of objects and events of every conceivable kind. Moreover, this complex of things, actions, and events does not come ready-labeled. The world is not like a museum in which things of a kind are put together for our convenience. Instead, they are scattered about in space and time, and we must be ready to identify them wherever they are found. But identification involves mental activity. What is the character of this mental activity and why is it aroused?

The simplest form of identification is recognition. Psychologists inform us that recognition is often imme-

[1] Sidgwick, *The Use of Words in Reasoning*, p. 150.

diate, that an object arouses associates, and these give the sense of familiarity. "Part of the recognition of an object that is not handled or that does not give rise directly to movements is due to the fact that its uses are appreciated, that, when it is recognized, one knows at once what to do with it and how to use it."[1] Such immediate recognition is an unconscious classification. The character of the object is appreciated and the proper habits and attitude are aroused. Thus there is a practical aspect for identification. Similar things have similar properties: fire burns; this kind of tree bears delicious fruit; a silent baby is usually in mischief, etc. If we had to deal separately with each thing, we should not know what to do; it would be impossible to bring past experience to bear. We may say, then, that classification, unconscious or conscious, is the only way in which we can handle our complex world of individual things.

Types of Classification. But a *felt* identification is not sufficient for human needs. To handle things properly and commodiously mankind has had to work out methods and principles of grouping adapted to the purpose in hand. There are two main types of classification, the artificial and the natural, and these correspond to the dominant purpose. In an artificial classification, the purpose is to enable us to handle the facts in a given realm as quickly and easily as possible; that is, the purpose is practical. In a natural classification, the purpose is to allow things or facts to arrange themselves in accordance with their essential nature. Let us look a little more closely at these two types.

[1] Pillsbury, *Essentials of Psychology*, p. 209.

Artificial Classification. In artificial classifications where the purpose is facility in handling, some obvious and, as a rule, external attribute is selected as the basis. A good example of this type is the method by which books are catalogued in a library. The first letter of the author's name determines the place of the book on the shelves and in the catalogue. If this simple device is insufficient, the subjects are likewise arranged alphabetically. This mode of artificial classification is usually called 'index classification.'

Another kind of artificial classification is the so-called 'diagnostic classification.' The purpose at work here is the identification of an object. A doctor wishes to diagnose a disease so that he may know what procedure to adopt in his treatment of it; or a collector wishes to label a butterfly which he has found. In both cases the the search is made for characteristics which will indicate the group to which the object belongs. Nature-study books are full of devices for identification of plants and animals according to pretty obvious traits.

Natural Classifications. Natural classifications are, as we have said, based upon as thorough investigation of the nature of the things classified as is attainable at the time. The purpose is to find some arrangement which corresponds to our knowledge and reflects a kinship in the things themselves. Such an arrangement is a difficult feat to achieve, and the growth of science is traceable in the perfecting of systems of classification. It is practically always necessary to penetrate below the surface and to build on those underlying attributes which reveal themselves only to the eye of the trained observer. "Thus the classification of

birds, for instance, is based largely upon fundamental differences in anatomical structure. Birds, not as we see them, but as they are when stripped of plumage and in their nakedness, are the real objects of consideration in such a system of classification. The result is that in the same group there will appear side by side a number of birds whose surface markings are exceedingly disparate, such as the blue jay and the crow, or the English sparrow and the cardinal. It is always a broadening experience, as regards our habits of thinking, when we are able to discover some essential similarity at the basis of a marked surface dissimilarity."[1]

Natural classification is scientific classification and reflects all the purposes and methods of science. The ultimate aim is, of course, explanation, but there is also the subordinate aim of convenience. This combination of purposes is very well brought out by Jevons: "By the classification of any series of objects is meant the actual or ideal arrangement together of those things which are like and the separation of those which are unlike, the purpose of the arrangement being, primarily, to disclose the correlations or laws of union of properties and circumstances, and secondarily, to facilitate the operations of the mind in clearly conceiving and retaining in memory the characters of the objects in question."

Natural Classification in the Light of Evolution. During the eighteenth and the first half of the nineteenth century, scientists believed in the existence of natural kinds of species which were changeless in character and sharply distinct from one another. The task of the anatomist was to dissect these species and deter-

[1] Hibben, *Logic, Deductive and Inductive*, p. 58.

mine their essential or defining marks. The hope was to enumerate and describe these fixed species once for all, and then to erect a scheme of classification into which they would all fit. But continued investigation convinced naturalists that there was something scholastic and unreal in sharp distinctions; that species merged with one another on the edges. With the gradual rise of the theory of evolution it came to be recognized that there are genetic relationships running all through the organic realm and that species are, so to speak, abstractions which over-simplify nature. Species are not immutable, but, instead, shade over into one another. Classification is a tool which must not be read into the complex realm of things too naïvely.

Character of Systematic Classification. Particular classes are formed by grouping objects together according to their likenesses and differences. But a further task ensues, that of showing the relations between the classes thus formed. We are not satisfied to leave particular classes in isolation, but demand a system of classification in which classes are arranged in accordance with conceived relations of *coördination*, *subordination*, and *superordination*. Such systems must express an internal unity which is manifested in various ways in the subordinate classes. All the classes must have certain attributes in common, and these common attributes must not contradict the more specific attributes which differentiate the subordinate classes from one another. To put the matter less technically: classification works within some field of objects and seeks to arrange them by selecting, first, those attributes which they *all* possess, and, secondly, those groups

of further and more specific attributes which divide the whole field into related yet distinct classes. So far as possible the differences which divide the whole into parts must be seen in their relation to the fundamental similarities. A particular class is a species (difference) *of* (similarity) the genus. A good classification should make it possible to see any fact in the light of the whole. The following example brings out this presence of an underlying agreement manifesting itself in different ways:[1]—

Vegetable Kingdom

- Rootless, leafless, stemless plants (*Thallophyta*)
  - Seaweeds, etc. (*Algæ*)
  - Fungus (*Fungi*)
- Moss-like plants (*Bryophyta*)
  - Liverworts (*Hepaticæ*)
  - Mosses (*Musci*)
  - Ferns (*Filices*)
- Fern-like plants (*Pteridophyta*)
  - Horsetails (*Equisetaceæ*)
  - Club-mosses (*Lycopodiaceæ*)
- Seed-bearing plants (*Spermaphyta*)
  - Naked-seeded plants (*Gymnospermæ*)
  - Encased-seeded plants (*Angiospermæ*)

The student should examine the type of classification found in any good textbook in biology in order to familiarize himself with the mechanism of the process.

Classification and Division. There is a very close connection between classification and another logical process called 'division.' The main difference in the meaning of the two terms is this: that when we classify, we think of ourselves as moving upward from the indi-

[1] Quoted from Taylor, *Elementary Logic*, p. 45.

vidual things to schemes of arrangement according to likeness and difference; that when we divide, we think of ourselves as moving downward from the more general and inclusive modes of grouping to more specific sub-classes. Such a distinction is, however, rather artificial, for, in the solution of any real problem of arrangement, our thought moves in both directions at once. Yet, while recognizing to the full this double movement of thought, it still remains true that logical division is a method with a definitely conceived form which can be used as a test of any classification. As a method, it is inclined to be more doctrinaire and *a priori* than classification.

Technical Terms used in Division. Division is a process which may be carried through many stages. The general class with which it starts is called the 'summum genus,' that is, the highest genus. The subclass with which it stops is called the 'infimæ species,' that is, the lowest species. Between these two extremes, the terms 'species' and 'genus' are used as purely relative terms. In any series, the class above is the genus of the class below which is its species. The class directly above is the 'proximum genus' of the species just below. Where more than one species have the same proximum genus they are called 'coördinate species.' The coördinate species of a genus are called its 'constituent species.' The principle used in dividing is called the 'fundamentum divisionis,' or basis of division. It is that aspect of the summum genus which enables us to separate its constituent species. The *differentia* of a species is that characteristic which distinguishes it from its coördinate species.

Rules of Division. To be valid, a logical division must observe the following rules: —

1. A division must be complete. A complete division is one which exhausts the constituent species of the main class.

The necessity for this rule is at once apparent. The neglect of any species may easily lead to confusion in both theory and practice.

2. The constituent species into which the genus is divided must not overlap. A division is imperfect if an individual may belong to two species at the same time.

A few words in regard to these rules may be advisable. An *incomplete division* may be due to ignorance, lack of persistence, or to the special difficulties of the field. When the last is the case, it is sometimes met by forming a *miscellaneous* class to include obscure data. Such a miscellaneous class points to the tentative character of the classification which the division reflects. A violation of the second rule leads to *cross-division* and is the consequence of the employment of more than one basis of division. The division of the citizens of a country into the rich, the poor, and the educated is an example of cross-division.

Forms of Division. There are two forms of division, *dichotomous* and *classificatory* division. These are readily distinguishable. In dichotomous, or bifurcate, division the genus is cut into two classes, those which possess the character and those which do not. The one is a positive group, the other is an indefinite, negative group. Together, however, they are exhaustive of the genus. This process can be further continued on either of the two groups. The other form of division,

which I have called 'classificatory,' is the reverse aspect of any system of classification. The species into which the genus divides are positive and coördinate; their number is determined by the nature of the field. We may speak of such division as 'analytic' and of the corresponding classification as 'synthetic.'

Dichotomous Division. While dichotomous division is simple and exhaustive, it easily becomes unwieldy when carried far. Moreover, one side is negative and indefinite. All that is known about it is that it lacks the positive character the other side possesses. Under certain circumstances, however, the lack of a quality may be as interesting as its possession. Thus the division of the population into voters and non-voters, or of the working-classes into skilled and unskilled, gives us two groups which are really coördinate. Where we are interested in some part of the field rather than in the whole, tracing by dichotomy may be the easiest way. The following example, usually called the 'Tree of Porphyry,' makes the method clear: —

Substance
- Corporeal
  - Animate
    - Sensible
      - Rational
        - Mortal
        - Immortal
      - Irrational
    - Insensible
  - Inanimate
- Incorporeal

This method of division may be made instrumental to the working-out of a definition of a term. Thus, accepting the adequacy of the above *tree*, we can define man as 'a corporeal substance, animate, sensible, rational, and mortal.' Another example will serve to

bring out this use sufficiently. "The term to be defined is 'tuber'; the genus to which it is to be referred is 'stem.'

Stem
creeping — not creeping
underground — not underground
much thickened — not much thickened
possessing leaf-buds in the form of 'eyes' — not possessing leaf-buds in the form of 'eyes'

"In this division, we reach, as our definition of a tuber, 'a stem creeping underground, much thickened, and possessing leaf-buds in the form of eyes.'"[1]

Classificatory Division. This form of division is, in its final expression, the same as a system of classification. The species under any genus must be coördinate and exhaustive. It should be remembered that the movement in division is downward from the general to the more particular, while in classification it is upward from the particulars to the more general. Only one other point needs, I think, to be mentioned. The *fundamentum divisionis* which is selected depends partly upon the purpose, partly upon the character of the subject-matter.

Dangers to be guarded against. Imperfect classification and division lead to the identification of things and ideas which should be discriminated. This is, as we have seen, the chief cause of ambiguity. It is, in fact, impossible to treat definition and ambiguity apart from the consideration of classification and division, for in these processes we are studying the methods by which

[1] Joseph, *An Introduction to Logic*, p. 113.

the mind orders and arranges its material. For the majority, an examination of the systems established in the various sciences is of less value than the critical analysis of less formal and better-known subjects. The chief danger to be guarded against is the merging together of things which should be distinguished. Natural law should not be confused with moral law, nor this with statute law. Education should not be identified with erudition, nor cleverness with talent. 'Democracy' should not be used as a blanket-term to cover representative institutions and the absence of caste. The proper use of terms is the result of the critical discrimination of different meanings, and these meanings themselves rest upon the keen perception and exhaustive treatment of likenesses and differences. After classification comes definition which is its fruition.

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CHAPTER VI

PRINCIPLES OF DEFINITION

Why Definition is Needed. In a former chapter we saw why language is a necessary instrument of thought and what some of its imperfections are. "The logical ideal of the relation between words and thoughts is, of course, that every thought should have its own name, and every name should have its own meaning."[1] This ideal is unrealizable because language is always changing. New shades of meaning are constantly being developed and old distinctions falling into disuse. But ignorance and mental indolence are also at work to make the use of words unclear. The correct employment of words depends upon three things: a large vocabulary, a fund of clear meanings, and the proper adjustment of the two. Unless these three things are forthcoming, there will be confusion in the individual's mind and misunderstanding when he attempts to communicate his ideas. Probably John Locke has pointed out these dangers, which only definition can correct, as well as any one:—

"For he that shall well consider the errors and obscurity, the mistakes and confusion, that are spread in the world by an ill use of words, will find some reason to doubt whether language, as it has been employed, has contributed more to the improvement or hindrance of knowledge amongst mankind. How many are there, that,

[1] Davidson, *The Logic of Definition*, p. 2.

when they would think on things, fix their thoughts only on words, especially when they would apply their minds to moral matters; and who, then, can wonder if the result of such contemplations and reasonings about little more than sounds, whilst the ideas they annex to them are very confused and very unsteady, or perhaps none at all; who can wonder, I say, that such thoughts and reasonings end in nothing but obscurity and mistake, without any clear judgment or knowledge?"[1]

We have said that definition can be of great assistance in overcoming such confusion and misunderstanding. But definition is a *process* which works with words and ideas. The individual must be capable of handling thousands of concepts and of distinguishing nice shades of meaning; he must also have a fairly large fund of knowledge. On the verbal side, he must be able to discriminate the uses of words. These are really two aspects of the same process, for I very much doubt whether the Mrs. Malaprops are much more sensible in their use of ideas than in their use of words. No one can make nice discriminations between words who cannot do the same for ideas. Both acts are achievements which do not come without effort and the establishment of good mental habits. And it is the belief of the logician that the study of logic should assist in the formation of such habits. It stresses the need and gives the general rules and methods which should be followed.

The Purpose and Nature of Definition. Definition has a double aspect. On the one hand, terms are defined in order to test correspondence in the use of words. Two disputants wish to discover, for example,

[1] *Essay concerning Human Understanding*, bk. III, chap. XI.

whether they have the same understanding for a pivotal word. If they do have, they can profitably proceed in their discussion; if they do not have the same idea, they must go further back until they find some common ground of agreement. On the other hand, definition arises as the temporary completion of a field of investigation. A definition is a summary in which ideas are ordered in relation to one another. A definition is a product just as the meanings concerned are products. Looked at in this genetic way, definitions are seen to be tentative.

There are two aspects to every definition because both ideas and words are involved. Ideas must be achieved and analyzed, and the meaning obtained must be attached to a word as *its* meaning. According to the setting, either of these aspects may be emphasized more than the other. Logic has recognized this possibility in the distinction between *real* and *verbal* definitions. When the *purpose* is, predominantly, to indicate how we intend to use a term, the result is a verbal definition. Dictionary definitions are of assistance here because they offer the average or common meaning. It must be remembered that the peculiar purpose or point of view of the individual enters in to qualify the average meaning, which is in many ways an abstraction. Verbal definitions represent past achievements. When, on the other hand, we are interested primarily in getting an adequate idea, we are said to achieve real definitions. We try to get behind words to the things and concepts for which they stand.

To define is, then, to indicate the meaning of a word by achieving an analyzed concept. Definition is an at-

tempt to attain to a definite meaning. On the mental side, it is largely an outgrowth of classification and division.

Logic stresses General Terms. Logic has been so impressed by scientific method that it has connected definition with classification and division. It is for this reason that writers in logic usually define definition as the process of determining the connotation of a term. It will be remembered that general or class terms have two aspects, the things to which they apply and the attributes which they imply. To define such a term is to state the attributes characteristic of the class and, indirectly, to establish its denotation.

It is because vague terms become ambiguous that the need for definition is felt. We wish to know exactly what a term stands for. For example, is a 'communist' the same as a 'communard'? Now, in the case of proper names the ambiguity can be overcome by pointing to the person meant or by adding descriptive words to make the reference definite. Much the same process is necessary for abstract terms which are simple. If a man asks me what 'redness' means, I can only reply by calling his attention to some red object and indicating the quality. I must assume his ability to abstract the quality and make it an object of thought. But the application of class terms cannot be indicated in this simple fashion. It is true that I can illustrate the use of such a term by an example or specimen. "But any example might illustrate a variety of things; if two persons, each of whom was entirely unacquainted with the language of the other, should try to communicate by pointing to objects to indicate the meaning of the words they were

using, they would illustrate the uncertainly of this method in its extreme form. If one of them should point to a horse, he might mean any one of a dozen different things: horse, or simply animal, or useful animal, or large object, or gray, or beautiful, or dangerous, and so on."[1]

Definition and Classification. It can readily be seen that real definition, or the process of securing clearer and more adequate ideas, is bound up with the logic of classification. To obtain a clear idea of a class we must see it as a part of a system. Thus the definition of a triangle, 'as that section of a cone which is formed by a plane passing through the vortex perpendicularly to the base,' reveals the triangle as related to a system of geometrical objects. The more scientific a definition, the more it is the reflection of a classified field. Definition is possible only when there is organized knowledge. In a very true sense, the whole of logic concerns itself with the method of obtaining well-defined ideas.

The Verification of Meaning. The verification of the meaning of a term is partly a social process, partly a process of investigation of the things, actions, or events indicated by the term. There is always a rough approximation of the meaning with which to begin. The problem is to narrow down and define the use. Take, for example, such words as 'freedom,' 'liberty,' 'justice,' and 'equality.' These words are often on people's lips, yet they are used vaguely. They refer to the same social field, but stress different aspects of that field. They are allied while at the same time distinct.

[1] Jones, *Logic, Inductive and Deductive*, p. 58.

What must be done in order to verify their proper meanings?

There are two levels in the verification of meaning. The first is to determine how the word is actually used; the second is to advance to a scientific concept which fits into a thoroughly classified and analyzed department of investigation. The first level gives us a clear *popular concept;* the second, if successful, gives us a *scientific concept.* It goes without saying, however, that the scientific concept is essentially a summary of the total field under investigation. It is difficult, if not impossible, to get a clear vision of its contents and implications apart from an intimate acquaintance with the field. Let the reader pick up a book on a subject with which he is not familiar and attempt to understand the technical terms while neglecting the facts and argument with which they are connected, and he will understand that the verification of scientific concepts is one with scientific investigation itself.

But logic is also concerned with a more general play of the mind which works for clear meanings whether at the level of common sense or at that of science. It can never be amiss to run over the denotation of a term and to discuss with others their understanding of its exact shade of meaning. The more complex the field, the more valuable is this practice. This method of verification goes back to the Greeks, who made much use of it when they realized how uncertainly terms are employed by the majority of men. Socrates crystallized this procedure and it has consequently been called 'Socratic *induction*' and 'Socratic *dialectic*' according as stress is laid upon the running over of examples or

upon the value of free discussion by means of question and answer. Every one should read some one at least of the earlier Socratic dialogues of Plato in which this suggestive method is used as a tool for deepening and defining the meanings of common terms. When this method is skillfully applied, one can see, as it were, meanings grow under one's eyes. Under such conditions, the definition becomes a living thing into whose secrets one is admitted. "The truth is — as most readers of Plato know, only it is a truth difficult to retain and apply — that what we gain by discussing a definition is often but slightly represented in the superior fitness of the formula that we ultimately adopt; it consists chiefly in the greater clearness and fullness in which the characteristics of the matter to which the formula refers have been brought before the mind in the process of seeking for it. . . . In comparing different definitions our aim should be far less to decide what we ought to adopt, than to apprehend and duly consider the grounds on which each has commended itself to reflective minds. We shall generally find that each writer has noted some relation, some resemblance or difference, which others have overlooked; and we shall gain in completeness, and often in precision, of view by following him in his observations, whether or not we follow him in his conclusions." [1] This method is especially advisable in the social sciences or in the handling of those general topics which arise so frequently for discussion. How few of us know exactly what we mean by such terms as 'nature,' 'justice,'

[1] Sidgwick, *Political Economy*, pp. 52–53; quoted from Minto, *Logic, Inductive and Deductive*, p. 90.

'law,' 'beauty,' etc.! Familiarity with the word deceives us into the belief that we understand the concept of which it is the sign.

Rules of Definition. The following rules of definition should put the student on guard against bad definitions and should give him some assistance in his attempts to formulate good definitions. All the technical terms needed have already been given in the chapter on "Division and Classification."

1. *A good definition is by proximate genus and essential difference.* We should try to find a larger class with which to connect the thing defined and then analyze and state the features of the thing defined which distinguish it from the other members of this larger class. By relating it to this larger class, we in a measure explain it, for we connect it with something more familiar. We are then in a position to pass down to important differences. This rule of definition brings out its intimate connection with classification and division.

When the difference offered is not essential or is vaguely conceived, we have an imperfect definition. Many imperfect definitions are suggestive because they reveal the fact that the topic concerned has not yet been clearly analyzed and classified.

2. *A term should not be defined by means of itself or of words which are synonymous with it.* We should avoid tautology. Dictionaries transgress against this obvious rule only too frequently. It is hardly enlightening to be told that life is a 'vital force,' or that pleasure is 'agreeable sensation,' or that altruistically is 'in an altruistic manner.'[1] More serious cases of defining

[1] For other examples see Davidson, *The Logic of Definition*, p. 62.

a thing by itself are the following: 'Life is that which distinguishes living from non-living things,' 'A cause is the invariable and *unconditional* antecedent of a phenomenon.' Relative terms are often defined in terms of each other. The consequence is that we go from one to the other and then back again and are not greatly advanced by the process.

3. *A definition should be neither too broad nor too narrow.* It should not cover a larger field than usage warrants nor a narrower field. Thus, to define a square 'as a rectangle,' or a root as 'that part of a plant which grows underground,' would be cases of overbroad definitions; while to define a dog 'as a domesticated animal used to guard the home' would obviously be too narrow. This rule is really only a development of the first rule, for, had the genus and essential difference been worked out, the resultant definition would have been neither too broad nor too narrow.

4. *A definition should not be expressed in obscure or figurative language.* The *purpose* of a definition should be kept clearly in mind. Where the field is necessarily technical, the use of technical terms is inevitable, and these must not be considered obscure however little they are understood by the layman. "The scientific definition of life as the dynamic condition of an organism would not be that of the plain man. The plain man thinks he knows what ivy is, and is not much wiser for being told that it is an epiphytic plant of the genus *hedera*. But he, too, may learn something from the definition of sickness as Nature's protest against the misdirection of her forces, or from St. Austin's phrase, 'virtus est ordo amoris'—the orderly and

progressive unfolding of love."[1] In this connection, Dr. Johnson's famous definition of network, as "anything reticulated or decussated at equal distances with interstices between the intersections," is usually given as a horrible example of misapplied learning.

5. *A definition should be expressed in positive terms whenever possible.* A definition should tell what a thing is, rather than what it is not. But ultimate conceptions can often be suggested and indicated by negative terms. Thus, a geometer may define a point as that which has neither length, breadth, nor thickness. We must, however, stress the meaning of a definition instead of its form. Where the concept defined is, itself, essentially negative, there must be a negative element in the definition. 'Liberty is absence of restraint.' 'Uncertainty is the want of a fixed mental attitude.' 'A bachelor is an unmarried man.' 'Injustice is the not keeping of covenant.' These are all negative in form, but as positive as their contents permit.

Other Forms of Definition. We have stressed what may be called the ideal form of definition, that through genus and differentia. In the Latin terminology of logic it is called definition *per proximum genus et differentiam vel differentias.* It is, as we have constantly pointed out, the reflection of a system of classification. When such a basis cannot be appealed to, resort must be had to other methods of making the meaning of a term clear. The methods usually adopted in such cases are as follows:—

1. *Description.* Individual things or events can be described in such a way as to make the application of

[1] Macleane, *Reason, Thought, and Language,* p. 175.

the term clear. It is possible to select outstanding properties or traits which serve to distinguish one thing from another. Accurate word-painting is an art which can be used to reproduce in the mind of the hearer or reader the essential characteristics of the scene or person described.

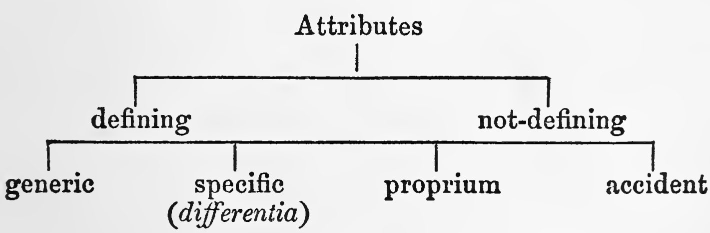
2. *Examples.* If well-selected examples of the term under discussion are given, it is possible to get a popular concept, at least, of the term. What does 'good' mean? How natural it is to run over examples of good men! In doing so, we are trying to pass from the denotation to connotation, a part of the denotation being less doubtful than the exact marks which make a man good. It depends upon the *purpose* how far the use of example is satisfactory. If this be practical, examples may meet the need and thus serve as a sort of definition.

3. *Genetic formula.* It is frequently important to know how a thing is made. Chemical formulæ do not tell us the essential properties of the thing unless they reveal to us a scheme of classification connected with such properties. They are, however, of distinct utility. Recipes, diagrams, instructions are practical definitions of a genetic kind.

The need lying back of definition is to understand the application of a term. Whatever meets this need is, in the broadest sense of that term, a definition. Hence the logician must not be dogmatic and set up only one type of definition. Purpose and setting must be taken into account. "In defining we look to what appears to be, for our immediate purpose, some striking feature. We try to define everything by something

better known. A fox would not be defined in the same way by a huntsman and by a naturalist. Death, which to the physician is the cessation of all vital functions, is to the singer of Hawthornden 'the thaw of all those vanities which the frost of life holdeth together.'"[1]

The Predicables. Logic is a very old science and has accumulated traditions and technical names. It is hardly justifiable to neglect these inherited distinctions, yet they must not be allowed to occupy too much space. Typical of this logical tradition are the 'Five Predicables.' *A predicable is literally that which can be predicated.* Now, only that which is a general term can be predicated of a subject. I can say that this object is a cat, but I can't assert of any other object that it is this particular object which I have just called a cat. In other words, I can't predicate an individual thing of another thing. What I can predicate is some *attribute* of the thing. But attributes are of two kinds, defining and not-defining. The result is that I discover the following division of attributes: —



These are the Five Heads of Predicables. In relation to the subject, the attributes found in the predicate may be generic, specific, differential, a constant property, and an accident. It is evident that these terms reflect classification and division, and this fact brings out once

[1] Macleane, *Reason, Thought, and Language*, p. 174.

more the intimate relation between the ideal form of definition and these processes.

The generic attributes give the connotation of the genus, while the specific attributes are those which are common to the species. It will be remembered that the differentia is that attribute, or attributes, which distinguishes one species from its coördinate species. But an object may possess, besides, some property not essential enough to be used as a differentia yet always present. When this property is deducible from the defining attribute it is called a 'proprium.' Attributes which sometimes belong to a class of objects and are sometimes absent are called 'accidents.' It is an accident that a university is located in one city rather than in another; that is, it is no part of the nature of the university to have this particular location. It is an accident of a thief to be in prison and of a politician to be honest. It should be borne in mind, however, that the term is here used in connection with classification and division. In another sense of the word 'accident,' it may well be that nothing is an accident; this means that every feature of an individual thing has a cause.

The Importance of Definition. From its very beginning, logic has stressed the importance of definition. It is impossible to think clearly and to avoid ambiguities without definite ideas and properly applied words. The very familiarity of words may lead us to suppose that we understand their use, and encourage us in the mere manipulation of verbal counters to the neglect of concepts. Verbal definition is important, but we must never forget that it rests upon ordered knowledge. Ordered knowledge is, however, only another name for

real definition. When we once realize this fact, we immediately become aware of the importance of definition, both as a process and as a result, and of the value to the individual of those mental habits of analyzing and distinguishing which give structure and clearness of outline to his accumulated knowledge. Definition is to the mind what discipline is to an army. Without discipline, an army is like a mob; it lacks both unity and the means which make unity effective. Without definition, the mind lacks differentiation and distinctness in its contents.

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CHAPTER VII

ASSERTIONS AND PROPOSITIONS

An Important Distinction. As logic has grown less formal and has been more influenced by psychology, it has more and more recognized the necessity of drawing a distinction between assertions and propositions. *An assertion is a living judgment which expresses the conclusion reached by an individual as a result of his thinking.* 'True patriotism stands for ideals,' and, 'I must take more exercise,' are for *me* genuine assertions which I understand. But when I glance at a list of exercises in a logic-text and come across the sentence, 'It is hot,' I am not absolutely certain what 'it' refers to. I tend to assume that the weather is referred to, but the writer may possibly have had a stove or a cup of tea in mind. Such isolated sentences need interpretation. The reader must make a judgment out of them by his own mental activity. And, as we all know, we are sometimes mistaken even when the context is quite complete. In contrast to personal, living assertions, then, propositions are sentences, or verbal complexes, which are supposed to symbolize such assertions; they are bodies into which some mind must breathe the breath of life. Propositions can exist on paper, while assertions can live only in the mind.

Under ideal conditions, the propositions put forward in actual cases of argumentation *may* be correct substitutes for assertions quite capable of conveying to other

minds the exact shade of meaning intended, but such is not always the case. Before this transference of meaning from one mind to another is possible, there must be painstaking effort on the part of the listeners to comprehend those who are speaking. How often a statement is misunderstood, or, as we are apt to say, twisted and distorted! The whole context, or *universe of discourse*, must be appreciated before a statement is given its correct setting. Such a context is present in any actual judgment, but it cannot be attached directly to the proposition. A sentence, spoken or written, cannot contain very much. Take the famous line in Shakespeare's *Henry VI:* "The Duke yet lives that Henry shall depose"; it is obvious that the ambiguity rests in our inability to get to the real assertion, assuming that there was one.

When conditions are very favorable, the distinction between assertions and propositions can be allowed to drop into the background. We suppose that we can pass from one to the other with ease. But when propositions are given piecemeal and apart from their context, it is often impossible to interpret them in any final sense. Under such conditions we are forced to pay attention to certain general aspects which should be clear. We change propositions into assertions as completely as possible, and then proceed to analyze them. But it is always well to bear in mind this important distinction.

Critical vs. Uncritical Assertion. Some mental processes are relatively unconscious, while others demand attention for their very being. We often act and speak habitually and imitatively. We do what we have

been accustomed to do and to see others do, and we say things which come into our heads or which we have heard other people say. The ideal of logic is to make individuals critical, and this aim is furthered by the knowledge it gains of mental processes and of scientific methods. But when logic became very formal and gave almost its entire attention to propositions taken separately, it became too mechanical to help the individual in his concrete thinking. What we shall try to do, then, is to explain these older analyses of propositions and give them a more vital setting.

Critical thinking seeks to determine the facts and principles of the case, to test both, and then to see whether the conclusion drawn is justified. It takes as little for granted as possible. It brings all the facts and principles into the open. This mental effort critically to interpret a case of reasoning which appears doubtful, leads to the marshaling of all our resources, to the bringing-out of all the facts and rules which bear upon the problem, to the analysis of likenesses and differences with other cases, to the more patient waiting for suggestive ideas. The result is usually a more adequate solution of the problem with a broader understanding of its meaning. Let me give an instance of critical *versus* uncritical thinking. A student comes to me and asks whether logic is a practical subject. I know pretty well by past experience what he means by the term 'practical.' But I ask him and he replies, 'Is it of use in everyday life?' Obviously, by everyday life he means business life. Will logic help him to succeed in his profession? It thus becomes a question of the *qualifications* of a successful business man. I may

deny that his ideal of success is a wise and adequate one, or I may try to show him that whatever makes his thinking more accurate and critical has a tendency to make him better qualified for business. The student has made certain assumptions as shown by the very words that he uses; in fact, he has revealed an outlook upon life.

What is an Assertion? We may define an assertion as *a mental act and content arising out of and concluding a process of interpretation.* Such an assertion is a judgment, a conclusion, a decision. Interpretations, generalizations, theories, inferences, beliefs, views, decisions are present from the early days of a child's life, and represent the growing, synthetic side of its mind. The growth of the mind, this putting two and two together, this recognizing and analyzing and comparing, is not a mechanical process. It is remarkable how soon the child notices things and seeks to interpret them. A little girl somewhat less than three years old was looking at her teddy-bear and noted for the first time the brown, hairless soles of its feet. The color struck her attention and she said, "That 's blood." Her brother, aged four, gave his judgment. "No, baby, that 's where the whiskers have come off." Baby replied, "We must buy some at the store and put them back on."

Of much of this continuous interpretation we are not reflectively aware. It does not stand out so clearly in consciousness that we remember it and can recall the various steps. The individual's field of experience seems to him always to have been much the same as it is now. Out there is the realm of physical things open to

inspection; and these things fall into kinds which are usually easily recognizable. In contrast to this *common world* of physical things is the more personal domain of the self with its feelings, valuations, and purposes; and there is, finally, the social realm of other minds. In other words, the mind of each individual is full of implicit assertions which cannot be accounted for apart from pretty constant mental activity. Interpretation is going on all the time and finds expression in judgments. These judgments are assertions which deal with these recognized realms. When I say, 'That 's Smith,' I mean that the person I see at a certain distance from me goes by the name of Smith. When I say, 'Nitrates are necessary for the soil in agriculture,' I take the physical world for granted and make an assertion about certain relations between it and plants. Thus we are interested in, and pass judgments about, the world in which we live.

Levels of Judgment. We have taken the term 'Judgment' in a broad sense as standing for both the process and the act of interpretation. Very often, the mental steps just previous to the assertion do not rise to the surface of consciousness. We feel certain that the person over yonder is Jones, but we don't just know why. He gives us the general impression of Jones. Only when doubt is cast upon our interpretation of our perception do we look more closely and analyze this feature and that which are connected in our mind with Jones. Perceptual judgments, or judgments bound up very closely with what we are perceiving, are of this almost intuitive character.

The more of a problem there is, the more reflective

is judgment. The process of reaching a conclusion now takes time, and we are aware of various suggestions and steps. We try to find exactly what is given and where the difficulty lies, and then ransack our experience to find a satisfactory solution. Such a mental process is called 'reasoning' and may take a long time; but when a solution is found, it appears as a judgment.

It is customary for logicians to classify judgments as 'dominantly perceptual' or as 'dominantly conceptual.' All judgments involve concepts, however, and there is no sharp break between the judgments of perception and the abstract judgments of history and of science. A huge body of facts must be collected, analyzed, compared, and organized before such abstract judgments are possible. In the latter half of the book we shall have much to say of these progressive steps which raise us to such abstract judgments as 'The planets travel in elliptical paths,' and 'Democracy is still in process of development.'

Concepts and Judgment. Judgments always contain concepts, or meanings. It was this fact that led many of the older logicians to think of judgments as formed from the union of two concepts. That is now seen to be too mechanical an idea. The content of a judgment is complex, but it is also unitary; it makes up one assertion.

The relation between concepts and judgment is a vital one and can be understood only when approached in a genetic way. Concepts are the material of judgment and are at the same time products of judgment. To ask which is prior in any absolute way is like asking the old puzzle, Which came first, the hen or the

egg? Judgments grow out of conceptual material, but this conceptual material is itself the deposit of other judgments. Again, this conceptual material is modified by judging as a process and exists in an unique way in any really new judgment. An example may make this clearer.

Take a person who is visiting an aquarium for the first time. Undoubtedly, he has some idea of many of the kinds of fish which are on exhibit, but his ideas are usually somewhat vague and unsettled. When he comes before the tank in which is, for instance, the devil-fish or ray, he makes the identification, and, in so doing, modifies his concept. It is no longer the hazy concept it once was. Again, take the student who comes to college. He has a general idea of what a college is for. But, as he progresses from the freshman to the senior year, he is compelled, by the very pressure of his complex experience, to pass judgments in which this general ideal is modified, deepened, and enlarged. Judgment is, in fact, the individual's system of concepts in process of adjustment and enlargement. Concepts drop into the background so far as consciousness is concerned and become potential or implicit when they are not the living parts of a judgment.

Judgment defined. There are many definitions of judgment, varying slightly in their form or their emphases from one another. Two aspects of every judgment stand out clearly. There is, first, the complex content which is held before the mind, and, second, the attitude of belief or acceptance taken toward it. Let us look a little more closely at these two aspects.

Every judgment involves a complex content. What

the content is depends upon the location of the problem which gives rise to it. It may be dominantly perceptual with just a touch of conception at the point which needed interpretation, as when I step outdoors of a morning and exclaim, 'It 's a fine day after all'; or it may be almost equally perceptual and conceptual, as when I classify a plant which I run across in my stroll through the woods; or it may be entirely conceptual, as in the statement of some law of nature. In every case, however, the total content which I hold before my mind is complex, the nature of the complexity varying with the field involved and the nature of the problem to which the judgment is an answer. Sometimes we are interested in the qualities of things, sometimes in classes and their relations to one another, sometimes in comparisons, sometimes in personages and events. The character of the content determines the *type of judgment* with which we have to deal.

Again, every judgment involves a mental attitude which is that of belief or assertion. As a rule, the psychologist prefers the term 'belief,' while the logician speaks of 'assertion.' The content is believed or asserted to hold of the realm which is under discussion. When we use the term 'belief,' we keep a more personal reference. 'It is my belief, or my judgment, that Napoleon was a worn-out man by the time of the Russian campaign.' Here a complex idea concerning itself with European history of somewhat over a century ago is accepted as true. I feel constrained by the pressure of the facts to think of Napoleon in this way. This mental constraint which arises out of the facts, says the logician, leads me to make the assertion, 'Napoleon was a

worn-out man,' etc. Napoleon and the Europe of his day are objects before my mind's eye, and I determine to think of one feature, which has been open to question, in this way rather than in that. The rest of my knowledge furnishes the setting for this once moot point which I now take to be settled.

A judgment is, then, a complex mental content which is asserted or toward which we take the attitude of belief. Such an assertion must be either true or false. It is this concern with truth or falsity which is distinctive of a judgment. In language, this characteristic finds expression in the indicative mood in contrast to commands, wishes, and questions. The reason for this difference rests ultimately in the fact that *knowledge is claimed or assumed in the one form, while not in the others.* When I ask a question, I request some one else to make an assertion, but do not myself make one. When I issue a command, I am concerned with an action which I am trying to bring about. In neither case am I expressing a belief.

All Knowledge Judgmental. We have stressed the fact that live judgments spring from problems. But such problems must always be local or concern points of doubt within a larger field about which there is, for the time being, no question. This larger field which furnishes the setting of the new judgment is itself, however, the product of the continuously accumulating judgments of the mind and the content of the new judgment is knit to it like new cells in a growing organism to the old. It is strictly true to say that all our knowledge has been, at one time or other, a part of the content of a judgment and that it is more or less im-

plied as a foundation for the new judgment. For this reason it must be ready to defend itself and to pass from a potential, or passive condition, to the center of the mental stage. Each of us possesses a system of concepts which has been the growth of years of guided and unguided mental effort. This system controls our thinking and is in turn controlled by it. Only a small part of it can appear overtly in the field of attention at any one time, but we have good reason to believe that all of it is there in a more or less effective way.

It is this implication of a larger setting for any new judgment, a setting which is usually more implicit than explicit, that the philosophical logician always stresses. And it is well to remember that logic is, after all, a philosophical discipline. I make no apology, therefore, for the introduction of the following quotation from Bosanquet: "We may then sum up so far: our knowledge, or our world in knowledge, exists for us as a judgment, that is, as an affirmation in which our present perception is amplified by an ideal interpretation which is identified with it. This interpretation or enlargement claims necessity or universality, and is therefore objective as our world, i.e., is what we are *obliged* to think, and what we are *all* obliged to think. The whole system in process of construction, viz., our present perception as extended by interpretation, is what we mean by reality, only with a reservation in favour of forms of experience which are not intellectual at all. Every judgment then affirms something to be real, and therefore affirms reality to be defined, in part, by that something. Knowledge exists in the form of affirmations about reality. And our world as existing for us in the medium

of knowledge consists, for us, of a standing affirmation about reality." [1]

It is for theory of knowledge to decide the character of knowledge and its relation to reality, while it is the function of metaphysics to determine the nature of reality. Logic takes both knowledge and reality for granted, and investigates the empirical question of *how* knowledge is actually built up and sustained by human minds. The conclusion which is forced upon us is that all knowledge is judgmental and is that which we are constrained to think by the pressure of our experience.

The Part played by Language. The actual operation of judging is often hard to recover. The modern logician, working hand in hand with the psychologist, is more and more convinced that this operation is not truly reflected in the verbal form which is called the proposition. So far as consciousness goes, the mental operation is usually simpler than the verbal form. Language is more analytic and detailed than the thinking of any one moment. There are two reasons for this. Language is used for communication and reflects all the distinctions painstakingly made in the past. The other reason lies deeper, language is a mechanism and, while instrumental to thought, is not identical with it. This disunion came out clearly in several of the preceding chapters, especially in the chapters entitled "Ambiguity and Language" and "Principles of Definition." It would be impossible for any mechanism to equal the fluidity and organic unity of thought. We must not take the skeleton for the living animal. Consciousness is a qualitative com-

[1] *The Essentials of Logic*, p. 32.

plex in which purpose, context, perception, and the whole apperceptive background of accumulated experience somehow coexist in a forward movement of interpretation. Language, on the other hand, is spread out in space and time, and necessarily adapts itself to this process of extension. *The purpose of language is not to reflect the actual genesis of a thought, but to express the result in a comprehensible way.* An argument should be capable of explicit formulation in such a way as to give the conclusion in relation to its grounds. So far as logic is a science of proof, it deals with the overt and not with the hidden; and the overt must find expression in language.

The Logical Treatment of Propositions. Modern logic tries to be as concrete and flexible as possible. It insists upon a study of the context to determine the point of view of the person who enunciated a given proposition. But within such a setting, or, as it is called, *universe of discourse*, a proposition or group of propositions is a bit of supposed knowledge which has to stand by itself as does a work of art which has once left the *atelier* of the master for the gallery. The proposition, as treated by the logician, is an instrument of analysis much as a sensation is for the modern psychologist. The student must be warned that the traditional analysis of the proposition to which we now pass is dominated by the mechanism of the syllogism and the idea of class relations which finds expression therein. For this reason, the 'logical form' of propositions is often a procrustean bed for propositions which do not naturally express class relations. The art side of logic will come to the fore in the next few chapters.

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CHAPTER VIII

THE LOGIC OF PROPOSITIONS

A General Division of Propositions. Propositions are divided into 'categorical,' 'hypothetical,' and 'disjunctive.' A proposition is said to be categorical when it makes a direct assertion. In it a predicate is denied or affirmed of a subject. 'The Romans were a strong race,' and 'Dead men don't bite,' are examples. A hypothetical proposition combines a condition with a consequent. If the condition is fulfilled, the consequent is held to be true. 'If the great war continues, the nations will be near bankruptcy,' and 'If there is an early frost, the peach crop will be spoiled,' are cases in point. A disjunctive proposition asserts that one of two or more alternatives is true. 'Man is either mortal or immortal,' and 'This rock is either marble or quartz,' are disjunctive in character.

The categorical type of proposition with its direct assertion is the simplest of the three forms and is always examined in logic before the others. It is the basis of the categorical syllogism. We shall take up the other types for discussion afterwards.

The Quality and Quantity of Propositions. Categorical propositions are classified according to quantity and quality. As regards quantity, they are either singular, or particular, or universal. *The quantity of a proposition is an aspect of the subject term.* If the subject denotes only one individual, the proposition is

said to be singular; if it denotes all the members of a class, universal; if only some members of the class are denoted, particular. 'Abraham Lincoln was a sagacious statesman' is a singular proposition because the subject term denotes only one individual. 'Some vines bear edible fruit' is a particular proposition because the subject term covers only a part of the class. The word 'some,' or an equivalent, is the sign of a particular proposition. 'All citizens are entitled to a vote' is a universal proposition. Its sign is 'all' or 'no' ('none'), or an equivalent.

The quantity of a proposition is not always definitely indicated by the appropriate logical sign. When this is the case, the proposition is called 'indefinite' or 'preindesignate.' The first task of the logician is to determine the quantity of such an assertion, for a really indefinite proposition is ambiguous and can have no place in an argument until its ambiguity is removed. Since the mechanism of the syllogism stresses quantity, it is absolutely necessary to classify an assertion before it can be used as a premise. The following examples will show how frequently propositions are left with their quantity implicit and even uncertain: —

> 'Uneasy lies the head that wears a crown.'
> 'Every single day has its blessing.'
> 'All flesh is not the same flesh.'
> 'The race is not always to the swift.'
> 'Haste makes waste.'
> 'A man's a man for all that.'

It is necessary to penetrate beneath the verbal form to the meaning and decide the denotation of the subject term.

Quality, when applied to propositions, is a term which refers to their character as affirmative or negative. An affirmative proposition asserts a predicate of a subject, while a negative denies it. The distinction is too familiar and ultimate to require elucidation. There are, however, certain dangers in interpretation which must be guarded against. The mere presence of a negative word in a proposition does not necessarily make it negative in quality. Such a word may be a part of either the subject or the predicate term. Only when the negative element involves the assertion of a relation of exclusion between the component terms is the quality negative. Take the proposition, 'Those who do not use their opportunities are unwise.' This is affirmative in quality because a positive relation between the two terms is asserted. Whether the subject or the predicate is positive or negative does not affect the quality of the proposition.

Symbolic Classification of Propositions. For the sake of easy handling, logic has classified all categorical propositions into four kinds according to their quantity and quality. This treatment has the syllogism for its end in view. While it is artificial and one-sided, it is convenient and often leads to a better understanding of the import of the propositions. These four kinds are, 'universal affirmative,' 'universal negative,' 'particular affirmative,' and 'particular negative.' To symbolize these the vowels *A*, *E*, *I*, and *O* are used. *A* and *I* stand for the universal and particular affirmative respectively, and are taken from the Latin *affirmo*; *E* and *O* stand for the universal and particular negative, and are taken from *nego*. It should be noted

that the singular proposition is considered a universal. It can be treated as such in the syllogism. Taking S and P for the subject term and predicate term respectively, we have the following classification: —

Universal	Affirmative:	All S is P.	A
	Negative:	No S is P.	E
Particular	Affirmative:	Some S is P.	I
	Negative:	Some S is not P.	O

Reduction of Propositions to Logical Form. The reduction of the varied statements of ordinary life to a form corresponding to this symbolism is a practical exercise which has some value. How much it is very hard to tell. "The indirect use is to familiarize us with what the forms of common speech imply, and thus strengthen the intellect for interpreting the condensed and elliptical expression in which common speech abounds."[1] Such an interpretation is often difficult. For instance, it is not always easy to decide whether a sentence is particular or universal, affirmative or negative. Take such sentences as the following and decide into which of the four kinds they fall: —

'A flower is a beautiful object.'
'Few were saved.'
'Women are jealous.'
'Only ignorant persons hold such opinions.'

The reduction of statements to logical form meets with three classes of difficulties. The statement may be *ambiguous* because of the grammatical order of its parts or because of the character of certain of its terms; or it may contain a reference to time. Let us look at these three classes in some detail.

[1] Minto, *Logic, Inductive and Deductive*, p. 69.

The order of subject and predicate may be inverted. 'Wide is man's mind' is an example of such inversion. The problem is so simple that there is no need to dwell on it. Again, a clause may be separated from the word it modifies; for example, 'He jests at scars who never felt a wound.' In such cases the only rule is to penetrate to the exact meaning. Lastly, some compound sentences really contain more than one proposition.

In the second class come partitive, exclusive, and exceptive propositions. A partitive proposition is one which makes a statement about one part of a class and implies another statement about the rest. There is, then, a sort of double meaning. Such words as 'all . . . not,' 'some,' 'a few,' etc., involve this ambiguity and double significance. 'All these children are not of school age' means that some are not, and, probably, that some are. Accent plays some part here. 'Few are chosen' means that most are not chosen.

An exclusive proposition is one which is introduced by such words as 'only,' 'none but,' 'alone,' and the like. 'None but ticket-holders are admitted' is a typical exclusive proposition. This means that all those who are not ticket-holders are not admitted. Thus the reference is to those who do not belong to the privileged class. As so interpreted, it is an *E* proposition. But it may also be treated as an *A* proposition by interchanging subject and predicate: 'All who are admitted are ticket-holders.' The old predicate lies within the subject-class.

An exceptive proposition is one which makes a statement of all the members except certain designated ones. Such sentences are introduced by such expressions as 'all but,' 'all except,' etc. 'All but the very poor enjoy

liberty' is an example. Two statements are usually involved: 'All who are not very poor enjoy liberty,' and 'All who are very poor do not enjoy liberty.' The first of these is more certainly implied than the second. It is therefore best to restate the proposition dropping out the exceptive particle.

The third class of difficulties is technical in character. The logical form, '*S* is *P*,' is the statement of a relation between classes and does not express either time or modality. The customary way is to throw this verbal element into the predicate. Thus, 'The soldiers had retreated' becomes 'The soldiers are in the class of those who had retreated,' and 'It *may* rain' becomes 'Raining is an event which *may* occur.' When we are trying to bring out class relationships, we must not be affrighted by the clumsiness of the result.

The Distribution of Terms. The logical form stresses the denotation of the subject and predicate terms. If, in a proposition, an assertion is made about the entire denotation of a class, the term indicating that class is said to be 'distributed'; if it is made of only a part of the class, the term is 'undistributed.' The distribution of the terms in the four kinds of categorical propositions has been worked out and is, besides, pretty obvious. *The universal affirmative distributes only the subject term.* Thus in the sentence, 'All mammals are vertebrates,' the assertion is made of *every* mammal, but there is no corresponding reference to the whole class of vertebrates. We have the right to say only that *some* vertebrates are mammals. *The particular affirmative distributes neither the subject nor the predicate term.* 'Some men are loyal' is an *I* proposition, and

there is in this no reference to the entire denotation of either class. Turning to negative propositions, we find *that an E proposition distributes both terms.* Both terms are *entirely* excluded from one another, and this means that there is some knowledge about all the members of both classes, just enough to validate this exclusion. 'No man is perfect' informs us that not a single man is perfect, and that not a single perfect being is a man. Lastly, *an O proposition distributes the predicate and does not distribute the subject.* 'Some men are not honest' tells us something only about some men, but we must know something about all individuals who are honest in order to know that some men are totally excluded. Examining the distribution of the terms in the four kinds of propositions, we find the following rule: negative propositions distribute their predicates, while affirmative propositions distribute only the subject and this only when the proposition is universal.

The Graphical Method. The relation between the subject and the predicate, as regards their denotation and the distribution of each term, can be graphically represented for the four kinds of propositions by Euler's method. If each term be represented by a circle, the result is as follows: For *A*,—

S, the subject class, is seen to fall entirely within *P*, the predicate class, for the vast majority of cases. (Fig. 1.) Only in the case of definitions, in which the two classes coincide, do we find the relation that of Fig. 2. The shading shows the distribution.

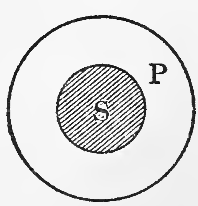

Fig. 1.

Fig. 2.

For *E*, —

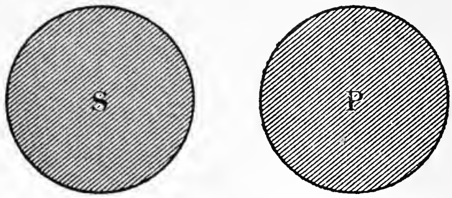

Fig. 3.

S falls entirely outside *P*. Both terms are distributed.

For *I*, —

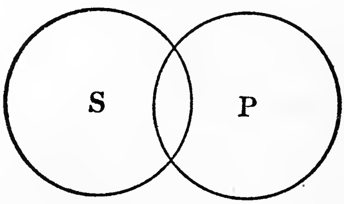

Fig. 4.

S and *P* overlap. Neither term is distributed.

For *O*, —

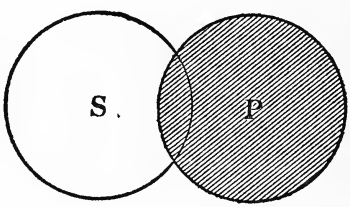

Fig. 5.

some *S* is known to be excluded by *P*. The predicate term is distributed.

Euler's method is applicable to the syllogism also. Hence it is well to become familiar with this graphical treatment of the denotative aspect of propositions.

The Purpose of Logical Analysis. This manipulation of sentences will undoubtedly strike the reader as artificial. When we say, 'All men are mortal,' we

do not mean that men are in the class of mortal beings. We think of the predicate in its connotation rather than in its denotation. But there is a purpose back of this stress upon denotation and class relationships, that of syllogistic analysis and testing. The 'logical form' is the syllogistic form. When the modern logician is not intent upon proof, he tries to understand the exact shade of meaning and the exact character of the relation present in a proposition. He seeks to classify propositions into natural types corresponding to levels of knowledge and reflection. But as a syllogizer, he is a specialist intent upon his method and bending everything to it. He then selects one aspect of propositions and moulds them in such a way as to bring out this aspect most clearly. 'All *S* is *P*' means that 'All *S* is contained in *P*.' It is the denotation of related classes that he has in mind. For a few chapters we shall be dominated by this purpose, and the student should bear this fact in mind if he cannot always see the importance of certain rather formal processes. The means secure their value from the end and are as valuable as the end. But it can further be said that the thorough study of the denotative aspect of categorical propositions cannot help but increase our understanding of their reach and meaning.

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CHAPTER IX

THE IMPLICATIONS OF PROPOSITIONS

Immediate Inference. Since the proposition is for the logician the unit of knowledge, he wishes to exhaust its meaning. He studies a proposition with the express purpose of finding out all that it implies. Now, it has been found that we can pass from one proposition to others containing the same two terms or their contradictories. Thus, 'All men are rational' enables us to say also that 'No men are irrational' and 'No irrational beings are men.' Such a passage from one proposition to others is called 'immediate inference.' The logician has usually been less concerned with the mental processes involved than with the validity and formal mechanism of the step.

Let us recall the distinction between a judgment and a proposition. A proposition is the verbal expression of a judgment. It is usually more stereotyped than the living judgment and practically always stresses some one aspect of the situation to the exclusion of others. Once a proposition is formulated, however, we can go back over it and analyze it, turn it about and see it from different angles. Thus, when I make the assertion, 'This book is on the desk,' I can afterwards note that 'The desk must be under the book.' Such a spatial relation has a double direction. Only one direction is brought out in the original proposition, but I realize that the other, also, exists. In like manner, as

we shall see, the logical form to which the logician reduces categorical propositions stresses a relation between terms and the denotation of these terms. The consequence is that other propositions can be seen to follow from any one formulation.

But because other propositions *can* be asserted on the basis of a given proposition, it does not follow that these other propositions present themselves without any mental effort on the part of the thinker. Immediate inference is not immediate in the sense that it is intuitive. The nature of the relations must be clearly grasped before such implications can be quickly drawn. It is for this reason that logical analysis is of value. Only one who has mastered this analysis can answer such questions as those given at the end of the chapter. Why, then, does the logician speak of the development of the implications of propositions as 'immediate inference'? Because he has in mind, as a contrast, the drawing of a conclusion from the putting together of two other propositions called 'premises.' In other words, the historical contrast is with the syllogism and its different mechanism.

There are *four* headings under which different kinds of immediate inference are brought: 'Opposition,' 'Conversion,' 'Obversion,' and 'Contraposition.' We shall now examine these in order and as briefly as is consonant with completeness and clearness.

The Oppositions of the Four Kinds of Propositions. Propositions are said to be opposed when they have the same terms as subject and predicate, but differ in quantity, or in quality, or in both. Within the setting given by the logic of propositions, we are able

to note certain relations of inclusion and exclusion which such statements bear to one another. The study of these relations will give us a better idea of the reach of each kind of proposition. It will likewise teach us how to establish each kind and what its denial implies.

If the symbols of the four kinds of propositions be placed at the four corners of a square with the universals at the top and the corresponding particulars at the bottom, we shall obtain the so-called 'Square of Opposition.' The sides and diagonals of this square represent relations between *A*, *E*, *I*, and *O*. Close inspection has revealed four kinds of relation among them:—

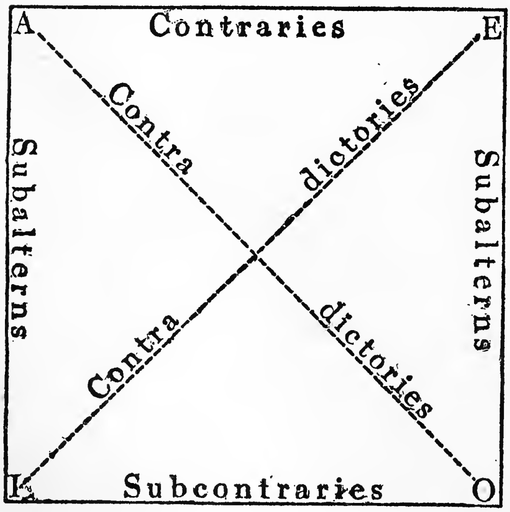

Fig. 6.

A and *E* are called 'contraries.' They differ in quality while the same in quantity, both being universal. If we examine them in relation to each other, we find that *both may be false while only one can be true.* Thus, 'All swans are white' and 'No swans are white' are contraries. They cannot both be true, yet they may both be false. As a matter of fact, we know that some swans are white while others are black.

A and *O*, *E* and *I*, are called 'contradictories.' They differ in both quantity and quality. Thus the members

of these couples are exact opposites. *One must be true and the other false.* 'All politicians are honest' and 'Some politicians are not honest' are contradictories. They cannot both be true nor both false. It follows that the assertion of *I* involves the denial of *E*, and the denial of *A* the assertion of *O*.

I and *O* are called 'subcontraries.' They agree in quantity and differ in quality. *Both may be true, but both cannot be false.* 'Some men are brave' and 'Some men are not brave' are subcontraries. Our experience tells us that both these propositions are true. When, however, we are able to state a universal, the one subcontrary is true, while the other, which is the contradictory of the universal, is false.

A and *I*, *E* and *O*, are called 'subalterns.' They agree in quality, but differ in quantity. Subalterns are both true when the universal is true. When the contrary universal is true, both subalterns are false. 'All men are rational' and 'Some men are rational' are subalterns.

It is a good exercise, to test the recognition of these interrelations, to start with the truth of any one kind of proposition and determine what follows for the other three. Granted *E*, *I* must be false as its contradictory, *O* must be true as its subaltern, and *A* must be false as its contrary. If we deny *E*, we automatically affirm *I*. It should be noted that common speech talks of contradiction whether we affirm the contrary or the contradictory of a given proposition. It is, however, convenient to have the two terms for these oppositions. Both are opposed to the given proposition, but the contrary is harder to establish than the contradictory.

Conversion. *Conversion is the process of interchanging subject and predicate while the quality of the proposition remains the same.* 'No man is perfect' becomes 'No perfect being is a man.' The original proposition is called the 'convertend' and the new one the 'converse.'

We must not forget that in conversion we are dealing with propositions which have been reduced to logical form and that the relation stressed is that of the denotation of the class terms. This relation is mutual. The situation is analogous to the arithmetical relation of equality between two quantities. If $A = B$, then $B = A$. If S is contained in P, P is to some extent contained in S.

The rule of conversion is that no term may be distributed in the converse which was not distributed in the convertend, The reason for this rule is obvious. A violation of it would involve making an assertion not justified by the denotation given in the original proposition. We should be going beyond our data.

There are two kinds of conversion: (*a*) simple conversion; (*b*) conversion by limitation, or *per accidens*. Simple conversion is the mere exchange of subject term and predicate term. *E* and *I* can be converted simply. Thus, 'No men are unaffected by self-interest' becomes 'No individuals unaffected by self-interest are men.' Both terms are distributed in the converse, but they were already distributed in the convertend. Hence the rule is not violated. 'Some men are honest' becomes 'Some honest beings are men.' Both terms are undistributed in the first as in the second proposition. *A* must be converted by limitation. Thus, 'All voters are

citizens' becomes 'Some citizens are voters.' In converting a universal affirmative, we are forced to pass from *A* to an *I*. It will be remembered that the predicate of an affirmative proposition is undistributed.

We have pointed out how *E*, *A*, and *I* can be converted. There remains *O*. But *O* cannot be converted because the result of an attempt would violate the rule. The undistributed subject term would become the predicate of a negative proposition and hence automatically claim distribution. The only way to interchange subject and predicate in the case of an *O* proposition is to obvert first and then convert. This double process is called 'contraposition.' Before we pass to it, we must examine obversion.

Obversion. *Obversion is the process of changing the quality of a proposition while retaining its meaning.* We may wish to state an affirmative proposition in its corresponding negative form, or *vice versa.* When this is the case, we must obvert. Which form is first used depends upon the purpose and psychological setting. But it is often convenient to restate the proposition so as to bring out another emphasis. The mechanism of the syllogism, also, makes it preferable at times to change the quality.

The method of obversion is to take the contradictory, or negative, of the predicate and then change the quality of the proposition. Its principle is that the denial of the contradictory of a predicate is the same as the affirmation of the predicate, and the affirmation of the contradictory the same as the denial of the original predicate. No mistake must be made in taking the exact opposite, or contradictory, of the predicate. The contradictory of

a negative predicate gives, of course, a positive term. Let us take, first, the symbolic logical form, 'All *S* is *P*,' and obvert in order to show the mechanism of the process. *P* becomes not-*P*, the quality is changed, and the proposition becomes 'No *S* is not-*P*.' The proposition 'None of the crew were saved' becomes 'All of the crew were individuals who were not saved,' which, rendered into good English, is, 'All of the crew perished.'

The following examples may make this discussion of principle and method clearer: —

A, 'All the people were rescued,' becomes, *E*, 'None of the people were lost.'

E, 'No practical men are poets,' becomes, *A*, 'All practical men are not-poets.'

I, 'Some vegetables are edible,' becomes, *O*, 'Some vegetables are not inedible.'

O, 'Some houses are not beautiful,' becomes, *I*, 'Some houses are ugly.'

The student must be on his guard against two things: misinterpretation of the original proposition, the obvertend, and the wrong treatment of the predicate.

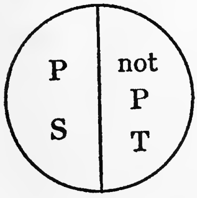

Fig. 7.

The principle of obversion can be represented graphically by means of a circle divided into two compartments. (See fig. 7).

"Then any given thing will fall into one or other of those compartments. If our proposition asserts that it falls into one, that is tantamount to asserting that it falls outside the other: the latter assertion would be the obverse of the former. *S* is *P*, implies that *S* is not not-*P*; *T* is not-*P*, implies that *T* is not *P*."[1]

[1] Jones, *Logic, Inductive and Deductive*, p. 122.

False Obversion. In valid obversion, the contradictory of the *predicate* is affirmed or denied, as the case may be, of the original subject. We have just noted the theory of such a change of quality. The universe is divided into P and not-P, and to affirm one is the same as to deny the other. This is the so-called 'law of counter-indication.' But this law does not hold of the subject in its relation to the predicate because the predicate is the larger term. If all S is P, it does not follow that no not-S is P, as can be seen from the use of Euler's circles.

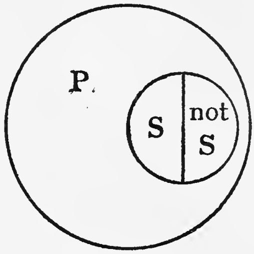

Fig. 8.

An example of false obversion may make this situation clearer. Take the proposition, 'All who are connected with this affair are honest.' Valid obversion would give, 'None who are connected with this affair are dishonest,' while false obversion would give, 'None who are not connected with this affair are honest.' It is obvious that the last proposition is not the equivalent of the other two.

Contraposition. *Contraposition is a process which combines obversion and conversion.* Thus the quality of the proposition is changed and its terms are interchanged. 'No voters are aliens' becomes by obversion, 'All voters are not-aliens (citizens),' and this, by conversion, becomes, 'Some not-aliens (citizens) are voters.' No new principle is raised by contraposition. The practical difficulty is to keep the first step clearly in mind while passing to the second. Inspection shows that the contrapositive has for its subject the contradictory of

the original predicate, while the quality is changed. This manipulation has value especially for the *O* proposition. It will be remembered that an *O* statement cannot be converted because of the lack of distribution of the subject. By obversion *O* becomes *I*, and this can be converted simply. For example, 'Some men are not inventors' becomes, successively, 'Some men are individuals who are not inventors' and 'Some individuals who are not inventors are men.'

Significance of Immediate Inference. The student can now better judge for himself the significance of immediate inference. Does he understand more completely the meaning of propositions when he can work out their implications? The processes are not hard to master and do repay attention. He who has mastered them does not have to *guess* whether one proposition is or is not equivalent to another. On the technical side, these processes are analogous to mental arithmetic, and, since they are always combined with the problem of reduction to logical form, give valuable discipline. The thinker must be in a position to handle propositions skillfully and correctly if he wishes to analyze and test his conclusions and those of others.

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CHAPTER X

THE SYLLOGISM AS A MECHANISM

The Nature of the Syllogism. The syllogism was the discovery of Aristotle, that is, he was the first to work out its theory and its rules. His own definition should therefore serve very well to introduce us to a brief study of its nature. In the *Prior Analytics*, he offers the following broad one: "Discourse in which certain things being posited, something else than what is posited *necessarily* follows on their being true." To syllogize is to reason things together and draw a necessary conclusion. But when we examine the Aristotelian syllogism more closely and see it in its context, we realize that this definition is too broad. There is necessary reasoning which is not syllogistic. What, then, is the *differentia?* This is to be found in the character of the things posited. *A syllogism is a discourse, or argument, in which three propositions are so related that one of them (the conclusion) follows from the other two.* The germ of the invention was the analysis of propositions into terms. The syllogism was conceived by Aristotle as a reasoning together of terms. His prime discovery was that whenever two propositions necessarily contain or imply a conclusion they have a common term, that is, only three terms between them.[1] In the last few chapters we have studied propositions separately, analyzing them, working out their implica-

[1] Minto, *Logic, Inductive and Deductive*, p. 170.

tions, reducing them to logical form. Now we must consider their interdependence.

A single proposition taken by itself is an assertion which does not give its ground. When it is advanced by a person other than ourselves, we are almost sure to demand why this conclusion is reached rather than another. In other words, we want the assertion connected with some generally accepted principle as its consequence before we are inclined to give it assent. This backward movement to more basic propositions is characteristic of the syllogism, and gives us a clue to its purpose: it is an instrument of testing rather than an instrument of discovery. I would not say that it never leads to new conclusions, but it is primarily a method of determining the consistency of propositions already thrown out for acceptance. There has been much misunderstanding on this point both now and in the past. John Locke was protesting against the idea, commonly enough held in his day, that men should do their creative thinking in terms of the syllogism, when he made the oft-quoted gibe, "God has not been so sparing to men, to make them barely two-legged creatures, and left it to Aristotle to make them rational." Even to-day there are not wanting psychologists who are at great pains to demonstrate that the syllogism is not an adequate description of how reasoning actually takes place in human minds. But any argument should be capable of being thrown into the syllogistic form or something analogous to it for purposes of testing its validity and assumptions. If the conclusion follows necessarily from other propositions, the argument is valid or self-consistent. *The conclusion need not, however, be*

true. Only if the other propositions, from which it is a valid deduction, are true, is it true.

Thus, the syllogism is an instrument which plays a part within a larger whole, that of experience and reflection. When we come to study the logic of science, we shall better understand whence the propositions come which appear in the syllogism. The main point to realize now is that the syllogism cannot prove their truth and does not try to; what it does try to prove, and can prove, is the consistency or inconsistency of their interrelations.

An Analysis of the Syllogism. Having got a clear idea of the nature and work of the syllogism, we can now describe it. It is a very simple bit of mechanism resting on the analysis of propositions into their logical form as assertions or denials of relation between the denotation of classes. This relation to which attention is directed is that of inclusion and exclusion. It will be recalled that an *A* proposition is symbolized thus: 'All *S* is contained in *P*.' Now, Aristotle saw that two terms could be combined in a proposition as a result of their relation to a third term. The following typical syllogism will illustrate this simple underlying idea:—

All men are mortal;
Socrates is a man;
Therefore Socrates is mortal.

The three terms are, 'Socrates,' 'mortal,' and 'man.' It is evident that 'Socrates' and 'mortal,' the two terms brought together in the conclusion, are established in their relation by means of a third term to which both are related. What is the character of this relation which holds between these terms and makes

them into a system? It is the relation of inclusion or exclusion, as the case may be, between their denotations. Applying Euler's circles to the above syllogism, we have —

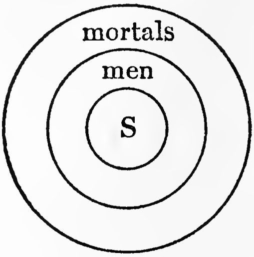

Fig. 9.

Aristotle's idea comes out clearly enough in the so-called '*Dictum de omni et nullo*' (statement concerning all and none) which is called the 'Axiom of the Syllogism.' "Whatever is predicated of All or None of a term is predicated of whatever is contained in that term." On the denotative side, this axiom is sufficiently expressed as follows: "A part of a part is a part of the whole." Thus, 'all men' is a part of 'mortals' and 'Socrates' is a part of 'all men.' This is as much of an axiom as the corresponding one of geometry.

The Elements of the Syllogism. We noted that Aristotle thought of the syllogism as a means of establishing a relation, negative or positive, between two terms by reason of their relation to a common third term. This common third term necessarily appears in each of the premises, but does not appear in the conclusion. The elements of the syllogism are, therefore, three terms and three propositions. The term common to the first two propositions is called the 'middle term,' and these propositions are the 'premises.' The conclud-

ing proposition is the 'conclusion.' The other two terms, which appear in the conclusion, are the 'extremes.' The subject of the conclusion, is called the 'minor term,' while its predicate is the 'major term.' These are the extremes mediated by the middle term. The premise which contains the major term is the major premise and is usually stated first; while the premise which contains the minor term is the minor premise.

The conclusion is the moot point, the problem or thesis in dispute. It is from it, therefore, that our thought starts. This fact shows that we are concerned in the syllogism with the mechanism of proof rather than with a movement of discovery. The question before our minds is, What premises will justify the conclusion? We are seeking the grounds of the conclusion. At this point we must note a certain difference in language form according as our thought moves downward to the conclusion from the premises or backward to the premises from the conclusion. In the first case, we use such terms as 'therefore,' 'hence,' etc.; in the second case, such terms as 'because' and 'since.'

The major premise is, characteristically, the statement of some broad general principle, some generalization from experience which can be applied to particular instances. It serves both to interpret and support the conclusion. The minor premise, on the other hand, points to the specific application which is in question. It is, characteristically, more factual than the major premise.

The Rules of the Syllogism. Having once comprehended the mechanism of the syllogism and related it to the logical treatment of propositions as relations

of exclusion and inclusion between classes, it is easy to formulate and to prove certain rules which no valid syllogism can violate. They are as follows: —

1. In every syllogism there should be three, and not more than three, terms, and these terms must be used throughout in the same sense.

2. The middle term must be distributed at least once in the premises.

3. No term ought to be distributed in the conclusion that was not distributed in the premises.

4. No conclusion can be drawn from two negative premises.

5. If one premise is negative, the conclusion must be negative.

6. No conclusion can be drawn from two particular propositions.

7. If one premise is particular, the conclusion must be particular.

The first rule follows from the mechanism of the syllogism. It is founded, as we saw, on the recognition of a relation between the major and minor terms because of their relation to a middle, or standard, term. The violation of this rule gives rise to the 'Fallacy of Four Terms.' If the four terms were clearly distinguishable, no one would try to syllogize them together. Thus, it would obviously be absurd to seek to draw a conclusion from such disparate propositions as, 'All men are mortal,' and 'Politics are improving in the United States.' But it sometimes happens that a term is ambiguous, there seem to be three when there are actually four terms.

The second rule brings out the condition without

which the middle term cannot perform its assigned function. Were both the major and the minor terms related to only a part of the middle term, we could

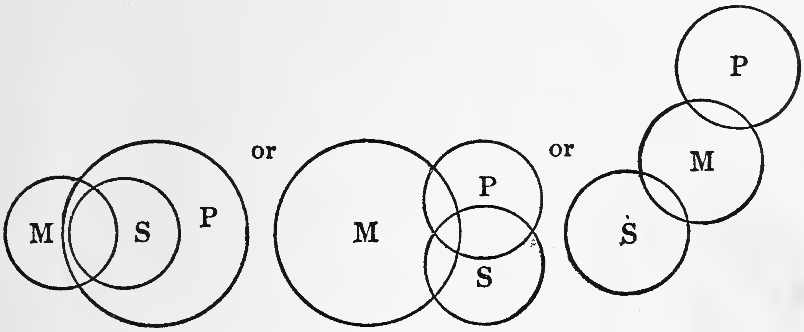

Fig. 10.

have no assurance that they were related to the same part. In other words, the information given would not tell us about the relative position of the terms to appear in the conclusion. This indefiniteness reveals itself in the relations of the circles representing the three terms. If in accordance with tradition, we let S, P, and M stand for minor, major, and middle term respectively, an undistributed middle shows itself in the existence of different possibilities. (Fig. 10.)

This is illustrated more concretely in the following argument: —

> All voters are citizens;
> These men are citizens;
> Therefore these men are voters.

Diagrammatically, this syllogism is unsettled. We are not able to determine a fixed relative position for the two classes, these men and voters. (Fig. 11.)

A breach of this second rule involves the fallacy known as 'Undistributed Middle.'

The basis for the third rule is the very nature of the conclusion as a logical sequence from the premises. We have no right to say more about a term in the conclusion than was said in its premise. This principle has already appeared in 'Immediate Inference' in conversion. A violation of the third rule in connection with the major term is called 'Illicit Process of the Major,' or, more briefly, 'illicit major'; its violation in connection with the minor term, 'Illicit Process of the Minor,' or 'illicit minor.' One example may serve for both.

All good citizens are voters;
This person is not a good citizen;
Therefore this person is not a voter.

The conclusion is a singular proposition and negative. It will be remembered that the predicate of a negative proposition is distributed. But this term is not distributed in the major premise. Hence this is a case of illicit major. This fallacy may be illustrated by circles.

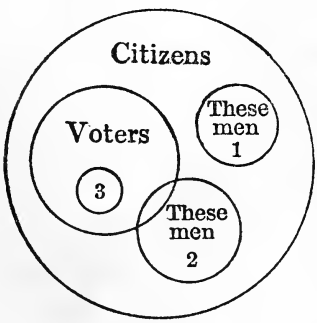

Fig. 11.

The fourth rule is understood as soon as we realize that two negative premises mean that there is no term by means of which we can locate the major and minor with reference to each other. Functionally, there is no middle, or mediating, term. We know that S and P both fall outside of M, but that fact gives us no clue in regard to their interrelation. This lack of mediation appears in the following argument: —

No bird is a bat;
This is not a bird;
Therefore this is a bat.

The use of circles may make the situation clearer.

Fig. 12.

What the rule points to is our inability to draw a conclusion from the information given in the premises. This *may* be a bat, but there are other possibilities. In applying this rule, we must be on our guard against propositions which are only apparently negative. Care in the analysis of sentences must precede the use of these formal rules.

The fifth rule is seen to hold as soon as we note that the two terms related in the conclusion do not have the same relation to the middle term. If one premise is negative, one of the extremes must be excluded in whole or in part from the middle term. But the other premise must be affirmative by rule four, and therefore asserts a partial or total inclusion in the middle term. This alliance with a term which excludes the other extreme involves it also as a party to the exclusion.

The sixth rule can be demonstrated by means of the preceding rules. Both premises cannot be particular and contain the necessary distribution. In the first place, only one of the two particulars can be negative (rule 4). There are left three possibilities, *I I*, *I O*, and *O I*. *I I* fails because it does not distribute the

middle term. *I O* and *O I* fail for another reason; they involve a negative conclusion (rule 5). But a negative conclusion means that the major term must be distributed in the major premise. But neither *O I* nor *I O* can distribute more than one term. Hence an argument having two particular premises is invalid because it commits either the fallacy of undistributed middle or that of illicit major.

The seventh and last rule can be demonstrated in much the same way as the sixth. If one premise be particular, there are three general possibilities, a combination of *A* and *I*, of *A* and *O*, and of *E* and *I*. If *A* and *I* have a universal conclusion, the minor term, the subject of the conclusion, must be distributed in the minor premise to avoid an illicit minor. But the premises together distribute only one term, and this must be the middle (rule 2). Bearing in mind the distribution of the terms in the four kinds of propositions, the student can easily prove why *A O* and *E I* cannot have universal conclusions.

The Formal Syllogism and Actual Reasoning. In the present chapter, we have treated the syllogism as a mechanism for testing the consistency of an argument. While such knowledge as we have gained is useful, there are vital objections to this purely formal view. In the first place, if the major premise is known to be true, the instance subsumed must already be known. We cannot know that *all* men are mortal without feeling that Socrates is mortal. There is no real advance in thought from the major through the minor to the conclusion. To this criticism of the syllogism, it is usually replied that the major premise is

really the statement of a *rule which we take to be without exceptions*, and the minor is the statement of an instance. If so, there is always a hazard in the syllogism, a point which we shall understand better when we come to the actual movement of scientific thinking. Another objection to the syllogism concerns the middle term. In an actual argument, how can you be certain that you have an unambiguous middle term? Take the following example: —

All men love good stories;
Smith is a man;
Therefore Smith loves this good story.

"Smith is in general a 'man,' and, therefore, loves good stories, but he is *not* a 'man' for the purpose of this particular conclusion; and so the sense of 'man' in the two premises is not the same, and this vitiates the argument."[1] This example brings out the fact that it is not always easy to apply a rule or, conversely, to know when an instance comes under a rule. Differences may be very important.

In actual reasoning, induction and deduction march together. Our knowledge of general principles develops as we apply them. There is an interpretation of rules by new facts and, at the same time, a reading of facts in terms of rules.

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[1] Schiller, *Formal Logic*, p. 200.

CHAPTER XI

THE FIGURES AND MOODS OF THE SYLLOGISM

The Figures of the Syllogism. The 'figure' of a syllogism is determined by the position of the middle term. Since the middle term may be subject or predicate in each of the two premises, four arrangements or figures are possible. Using the conventional symbols, we may state these four figures as follows: —

1. M is P	2. P is M	3. M is P	4. P is M
S is M	S is M	M is S	M is S
S is P	S is P	S is P	S is P

Inspecting these arrangements of the three terms in the two premises, we find that the middle term is the subject of the major premise and the predicate of the minor in the first figure. This figure was considered by Aristotle the most satisfactory and was therefore called the perfect figure. The middle term is contained in the major, and the minor in the middle term.

In the second figure, the middle term is the predicate of both premises.

In the third figure, it is the subject of both predicates.

In the fourth figure, it is the predicate of the major premise and the subject of the minor premise.

The Moods of the Syllogism. The 'mood' of a syllogism depends upon the quality and quantity of the propositions composing it. It will be remembered that there are four kinds of propositions symbolized by the

letters *A*, *E*, *I*, and *O*. If we leave out all question of validity, these letters may be combined in threes to make the possible moods of the syllogism. The sixty-four possible moods thus obtained are as follows: —

AAA	~~AEA~~	~~AIA~~	~~AOA~~	~~EAA~~	~~EEA~~	~~EIA~~	~~EOA~~
~~AAE~~	AEE	~~AIE~~	~~AOE~~	EAE	~~EEE~~	~~EIE~~	~~EOE~~
AAI	~~AEI~~	AII	~~AOI~~	~~EAI~~	~~EEI~~	~~EII~~	~~EOI~~
~~AAO~~	AEO	~~AIO~~	AOO	EAO	~~EEO~~	EIO	~~EOO~~

~~IAA~~	~~IEA~~	~~IIA~~	~~IOA~~	~~OAA~~	~~OEA~~	~~OIA~~	~~OOA~~
~~IAE~~	~~IEE~~	~~IIE~~	~~IOE~~	~~OAE~~	~~OEE~~	~~OIE~~	~~OOE~~
IAI	~~IEI~~	~~III~~	~~IOI~~	~~OAI~~	~~OEI~~	~~OII~~	~~OOI~~
~~IAO~~	(IEO)	~~IIO~~	~~IOO~~	OAO	~~OEO~~	~~OIO~~	~~OOO~~

Fig. 13.

Many of these mathematically possible moods are soon seen to be invalid. Thus, by applying rules 4 and 5, that no conclusion can be drawn from two negative or two particular premises, we can decide on the invalidity of those moods through which a line is drawn. *I E O* must also be discarded because it sins against rule 3; it is clearly a case of illicit major. We are left with *eleven* moods, some of which are valid in certain of the four figures and invalid in others. The reason for this variation in validity according to figure is the relation between distribution and the position of the terms. We must now ask ourselves what moods are valid in the first, second, third, and fourth figures respectively.

There are two ways of determining the valid moods of each figure: by inspection and by the establishment of special rules for each figure. We shall use both methods for the first figure and then only state the

special rules for the other figures, leaving it to the student to demonstrate them.

Taking the eleven moods which were left us after the first general elimination and arranging the terms in accordance with the first figure, we have the following results :—

A. All *M* is *P* *A*. All *S* is *M* *A*. All *S* is *P*	*A*. All *M* is *P* *A*. All *S* is *M* *I*. Some *S* is *P*	*A*. ~~All *M* is *P*~~ *E*. ~~No *S* is *M*~~ *E*. ~~No *S* is *P*~~
A. All *M* is *P* *I*. Some *S* is *M* *I*. Some *S* is *P*	*A*. ~~All *M* is *P*~~ *O*. ~~Some *S* is not *P*~~ *O*. ~~Some *S* is not *P*~~	*E*. No *M* is *P* *A*. All *S* is *M* *E*. No *S* is *P*
E. No *M* is *P* *I*. Some *S* is *M* *O*. Some *S* is not *P*	*I*. ~~Some *M* is *P*~~ *A*. ~~All *S* is *M*~~ *I*. ~~Some *S* is *P*~~	*O*. ~~Some *M* is not *P*~~ *A*. ~~All *S* is *M*~~ *O*. ~~Some *S* is not *P*~~
A. ~~All *M* is *P*~~ *E*. ~~No *S* is *P*~~ *O*. ~~Some *S* is not *P*~~	*E*. No *M* is *P* *A*. All *S* is *M* *O*. Some *S* is not *P*	

Fig. 14.

Applying to these possible moods of the first figure the general rules of the syllogism, we soon discover that *A E E*, *A E O*, *A O O*, *I A I*, and *O A O* are invalid because of an undistributed middle or an illicit major. Crossing these out, we have *A A A*, *A A I*, *A I I*, *E A E*, *E A O*, and *E I O* left as valid. Of these *A A I* and *E A O* are the same as *A A A* and *E A E* in the premises and have *weakened* conclusions; they have therefore no essential independent value.

This analysis of the possible moods of the first figure should illustrate the use made of distribution in the syllogism. An inspection of the valid moods leads to the discovery of the two following special rules for the first figure: —

1. The minor premise must be affirmative.
2. The major premise must be universal.

These two rules can also be demonstrated by means of the general rules of the syllogism. To prove that the minor premise must be affirmative, we use the indirect method of approach. Suppose it to be negative; then the conclusion must be negative (rule 5). But if the conclusion is negative, the major term must be distributed (rule 3). To distribute P, the major premise must be negative since only the predicates of negative propositions are distributed. But by hypothesis, the minor premise is already negative. Hence we should have two negative premises, which is impossible. It follows that the minor premise must be affirmative.

Having established this rule, we can easily show that the major premise must be universal. An affirmative minor does not distribute the middle term in the first figure. Hence it must be distributed in the major premise (rule 2). But only a universal will do this. Thus the second special rule follows.

A similar method is applicable to the three other figures. For the sake of completeness, the special rules for these figures will be given. There is no need to burden the memory with them, however, as in actual practice the general rules will be found sufficient as tests of validity.

The rules for the second figure are: —

1. The conclusion must be negative, and one premise must, also, be negative.
2. The major premise must be universal.

For the third figure they are:—

1. The conclusion must be particular.
2. The minor premise must be affirmative.

For the fourth they are as follows:—

1. If either premise be negative, the major premise must be universal.
2. If the major premise be affirmative, the minor must be universal.
3. If the minor premise be affirmative, the conclusion must be particular.

Reduction to the First Figure. We noted that Aristotle regarded the first as the perfect figure. He did so because the Axiom of the Syllogism has in it its simplest application. On the propositional side, the minor premise is subsumed under the major, which is a universal. The process of changing the other figures to the valid moods of the first is called 'reduction.' During the Middle Ages, when the syllogism held such a revered place, elaborate rules for this reduction were worked out and embodied in mnemonic lines. This process and these lines have had such a place in history that it would seem unwise not to mention them. There is an inimitable flavor of the past about them. Reduction is, moreover, a process which casts light upon the mechanism of the syllogism and connects it with conversion.

The following verses in scholastic Latin served the

double function of recalling the valid moods of each figure and of giving rules for reduction: —

> B*arbara*, C*elarent*, D*arii*, F*erio*que prioris;
> C*esare*, C*amestres*, F*estino*, B*aroko*, secundæ;
> Tertia, D*arapti*, D*isamis*, D*atisis*, F*elapton*,
> B*okardo*, F*erison*, habet; quarta insuper addit
> B*ramantip*, C*amenes*, D*imaris*, F*esapo*, F*resison*.

Unless one has a very facile memory, there is not sufficient value in the stanza to justify memorizing. It is interesting to see its construction and to realize how it was used. The vowels in the names give the moods. Thus, 'Barbara' is the *A A A* mood of the first figure. The words in genuine Latin indicate the figures. Suppose we select 'Disamis' in the third line. Since it is in the third figure we must write it in logical symbols as follows: —

> *I*. Some *M* is *P*.
> *A*. All *M* is *S*.
> *I*. Therefore some *S* is *P*.

The first letter of each mood of the 'imperfect' figures indicates the corresponding valid mood of the first figure to which it can be reduced. Thus, 'Cesare' and 'Camestres' of the second figure are reducible to 'Celarent' of the first. The letters *s*, *m*, and *p* also have their meaning for the process of reduction. Placed after a vowel, *s* indicates that the proposition represented by the vowel must be converted simply; *m* (*muta*) indicates that the premises must be interchanged; while *p* shows that the proposition after which it is placed must be converted by limitation, or *per accidens*. An example of reduction will make this clearer. Take 'Disamis' again. The initial letter indicates that 'Darii' is the corre-

sponding mood of the first figure. *I A I* of the third figure becomes *A I I* of the first by conversion of the major premise, interchange of the premises, and, finally, conversion of the conclusion to correspond to the transformation of the major term into a minor.

Comparative Value of the Figures. While the tradition in logic has favored the view that the first figure is somehow better than the others, a closer analysis has shown that they are independently valid and do not need reduction. They are independent types. In other words, the Axiom of the Syllogism holds primarily of the first figure and does not apply directly to the others. Different classes of arguments naturally fall into different forms. Thus, arguments involving the subsumption of an instance under a general principle take the first figure ; while those drawing negative conclusions from the absence of distinctive signs take the second. The following example of a second-figure syllogism will bring out this difference :—

> All fever-stricken patients are thirsty.
> This patient is not thirsty.
> He is not fever-stricken.[1]

A close inspection of this argument reveals the realization that if thirstiness is taken as a sign of fever, its absence is necessarily a sign of the lack of fever. It is for this reason that Minto speaks of the second as the 'Figure of Negative Diagnosis.' The third and fourth figures are of little value. It should be remembered that the third can give only a particular conclusion. For this reason it has sometimes been called the

[1] Minto, *Logic*.

'Inductive Figure.' The fourth is more of a *tour de force* and is not always admitted. Arguments do not naturally fall into it.

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CHAPTER XII

ABBREVIATED AND EXPANDED ARGUMENTS—EXTRA-SYLLOGISTIC ARGUMENTS

Enthymeme. It is seldom that all the parts of an argument are stated fully and explicitly. Some premise is taken for granted or is considered too obvious to justify formal statement. The thinker himself has not taken the time to analyze and formulate the grounds for his conclusion or he feels that his listeners wish only the salient points and can supply the remainder. In the first case, what he says reflects what has been uppermost in his mind; in the second case, he wishes to state only the necessary in order not to be wooden and pedantic. All this is natural and there is nothing in it which the logician has the right to condemn. Nevertheless, such condensation has its dangers against which the logician must issue his warning. Oftentimes suppressed premises are debatable and pass just because attention has not been called to them. They slink by, as it were, because they wear caps of darkness which hide their faces from scrutiny.

The logician does not wish to be the enemy of wit, but he can never forget that he is the guardian of reason. Brevity is the soul of wit and of the epigram, yet such syncopation must be capable of valid enlargement. It is of the character of such enlargement that the syllogism informs us. When a premise or the conclusion of an argument is not stated, the argument has the form of an enthymeme. The reasoning is not

fully expressed. *An enthymeme is, in brief, an elliptical argument.* A few examples will make this clear: —

'Death cannot be an evil, being universal.'[1]

'He is in love. He brushes his hat.'[2]

'She is a woman, therefore may be won.'[3]

Carlyle says of Sansculottism: 'It too came from God; for has it not been?'

It will be seen from these examples that literature is full of enthymemes. They add to the charm of a writer if not used too lavishly. On the other hand, they do not have so assured a place in arguments of a more didactic character as in science and philosophy. Not to leave things obscure is a merit in these fields.

Enthymemes are of the first, second, or third order according as the major premise, the minor premise, or the conclusion is wanting. The absence of the conclusion is a rhetorical device to make it more emphatic.

It is usually quite easy to supply the missing part of the argument. The two propositions which are given contain the three terms. Hence all that is needed is the perception of what proposition is absent and its construction. Thus the quotation from Goethe given above becomes —

> Whatever is universal is not an evil;
> Death is universal;
> Death is not an evil.

Prosyllogisms and Episyllogisms. Either premise of a syllogism may be justified by making it the conclusion of another syllogism. When this is done, the result is a complex argument in which one syllogism prepares the way for a second. The grounds of one of

[1] Goethe. [2] *Much Ado About Nothing.* [3] *Titus Andronicus.*

the premises of the final conclusion are offered. The first syllogism in such a complex is called the 'prosyllogism,' and the last the 'episyllogism.' Any connected argument of length involves the existence of interrelated syllogisms. The following is a good example: —

Prosyllogism	Everything which is able to restrain trade is a source of danger; Every monopoly is able to restrain trade; Hence, every monopoly is a source of danger.
Episyllogism	A company which has a complete control of a certain commodity is a monopoly; This trust has complete control of a certain commodity; Hence, this trust is a monopoly.

Final Conclusion. This trust is a source of danger.[1]

It will be noted that the conclusions of the first two syllogisms are the premises of the final conclusion.

Sorites. When many syllogisms are combined, as above, the combination is called a 'polysyllogism.' When all the intermediate conclusions are suppressed, a polysyllogism becomes a 'sorites.' A sorites is a chain or, literally, a piling-up of premises leading to a conclusion. The argument gathers weight as it proceeds.

Two types of sorites have been distinguished, the 'Progressive' or 'Aristotelian,' and the 'Regressive' or 'Goclenian.' The Aristotelian sorites moves from the least inclusive term to the most inclusive, from the subject of the conclusion to the predicate.

A is *B*. All Frenchmen are Europeans;
B is *C*. All Europeans are men;
C is *D*. All men are animals;
D is *E*. All animals are mortal;
A is *E*. All Frenchmen are mortal.

[1] Jones, *Logic, Inductive and Deductive.*

The Goclenian sorites moves in the opposite direction.

> *D* is *E*. All animals are mortal;
> *C* is *D*. All men are animals;
> *B* is *C*. All Europeans are men;
> *A* is *B*. All Frenchmen are Europeans;
> *A* is *E*. All Frenchmen are mortal.

A sorites can be expanded into a polysyllogism of which each unit is a syllogism of the first figure. Inspection will show the student that the order of each pair of premises must be changed to accomplish this expansion.

The following example of the use of the sorites in poetry is worth quoting to show how a logical form may be clothed upon and become a thing of beauty: —

> The longer life the more offence,
> The more offence the greater paine,
> The greater paine the lesse defence,
> The less defence the lesser gaine;
> The loss of gaine long yll doth trye,
> Wherefore come death and let mee dye.
>
> Come gentle Death, the ebbe of care,
> The ebbe of care the flood of life,
> The flood of life the joyfull fare,
> The joyfull fare the end of strife,
> The end of strife that thing wish I;
> Wherefore come death and let mee dye.[1]

Extra-Syllogistic Arguments. Not all forms of reasoning are syllogistic in type. For example, '$A = B$, $B = C \therefore A = C$.' We all recognize the cogency of this reasoning, yet we have no tendency to reduce it to syllogistic form. Instead, it is perceived to be an instance of an axiom which appeals to us as undeniable, namely,

[1] Sir Thomas Wyatt, 1503–44.

things which are equal to the same thing are equal to each other. The argument is in accord with the axiom, yet the axiom does not appear as a major premise in it. Deny the axiom, and you would be forced to reject the instance; but it is possible to accept the instance and seal its cogency without formulating the axiom.[1]

At one time there was a tendency to interpret such an argument as really syllogistic, as an enthymeme with an unformulated major premise, and to complete it thus: —

Things equal to the same thing are equal to each other;
A and *C* are things equal to the same thing;
A and *C* are equal to each other.

But inspection shows that our minor premise is not an exact reproduction of the original argument, which was that *A* and *C* are both equal to *B*. Even with this correction, we are not over our difficulties, for '*A* and *C*,' the apparent minor term, is not a term at all but two objects.

The truth is, that the syllogism is limited to the relations between classes in the way of inclusion and exclusion. So far as there is an axiom involved, such as the *dictum de omni et nullo*, this axiom is no more a major premise than the axiom, that things equal to the same thing are equal to each other, is an implied major premise for the mathematical argument referred to above. Deny either axiom, and, of course, the particular arguments fall to the ground. An examination of other types of arguments may make the situation clearer and enable us to realize that in the syllogism we are dealing with a comparatively simple sort of relation between classes. The point is, that in extra-syllogistic

[1] See Joseph, *An Introduction to Logic*, p. 273.

reasoning we have to do with different systems of relation, some of which are definite and some of which are indefinite. Only when the system is well enough known to justify a conclusion can we move freely within it.

Quantitative Relations

$$\left.\begin{array}{l} A \text{ is greater than } B \\ B \text{ is greater than } C \\ A \text{ is greater than } C \end{array}\right\} \text{Conclusion valid}$$

Relations of Direction

$$\left.\begin{array}{l} A \text{ is north of } B \\ B \text{ is north of } C \\ A \text{ is north of } C \end{array}\right\} \text{Conclusion valid}$$

In these cases, we move within a unitary system and can travel, as it were, back and forth in it. Each system has an axiom, but the axiom is founded on something of the nature of immediate perception. In the following instance the absence of such a unitary field of relations between things is immediately recognized: —

$$\left.\begin{array}{l} A \text{ is the employer of } B \\ B \text{ is the employer of } C \\ A \text{ is the employer of } C \end{array}\right\} \text{Conclusion invalid}$$

The study of extra-syllogistic reasoning is valuable because it brings into relief the character of the syllogism. It enables us to see that the subject-matter is fundamentally important.

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CHAPTER XIII

HYPOTHETICAL AND DISJUNCTIVE SYLLOGISMS

In a very real sense all propositions are categorical, for they all make assertions. But it has been customary for logic to distinguish propositions in which a predicate is directly asserted of a subject from two other forms of assertion, the hypothetical and the disjunctive. It will be remembered that these propositional forms were briefly defined in a previous chapter. They are of interest to us now because they appear as the major premises of two other kinds of syllogisms called, respectively, the 'hypothetical' and the 'disjunctive.'

The Hypothetical Syllogism. A hypothetical proposition combines a condition with a consequent. 'If this, then this,' is the general form. Thus, 'If it rains this afternoon, I shall not go out for a walk,' and 'If business is good, I shall buy an auto,' are examples of this form of assertion in which an *antecedent* is related to an inevitable *consequent.* A little reflection shows how extensive is this expression of a relation between two events. It is not too much to say that modern science is chiefly concerned with the discovery of relations. In practical life, again, we wish to know what to do in order to secure certain desirable results. If we can once find a rule, we have only to apply it.

In a hypothetical syllogism, the major premise is hypothetical, while the minor is categorical. The categorical minor either affirms the antecedent or denies

the consequent of the major premise. When it affirms the antecedent, the syllogism is said to be 'constructive,' or in the *modus ponens;* when it denies the consequent, the syllogism is 'destructive,' or in the *modus tollens.* An example of each type may make this distinction clearer: —

If the winter is really over, we shall have some warm weather;
The winter is really over;
Therefore we shall have some warm weather.

This argument is constructive because we affirm the antecedent and thereby have the right to affirm the consequent in the conclusion. In the following, however, we deny the consequent: —

If human life were considered precious, war would be judged a crime;
War is not judged a crime;
Therefore human life is not considered precious.

It should be noted that the distinction between affirmative and negative has no meaning for hypothetical propositions. We are either able to affirm a connection between an antecedent and a consequent or we are not able so to do. In the latter case we are not able to pass any judgment of this character. The most we can do is to state a probability. In a later chapter we shall study the nature of probability; but it would complicate things too greatly to introduce it here.

Another point needs attention. Either the antecedent or the consequent or both may be negative. In such a case the categorical minor may be negative in form and yet affirm the antecedent, or affirmative in form and yet deny the consequent. The way to avoid error is to get both parts of the major premise clearly before the

mind, and then to see them in their relation to the minor. In the following syllogism the minor is negative in form, yet it affirms the antecedent: —

> If there is not a strike at the works, the union has been disrupted ;
> There is not a strike at the works ;
> Therefore the union has been disrupted.

The Rule of the Hypothetical Syllogism. The rule of the hypothetical syllogism is sometimes spoken of as the 'Law of Reason and Consequent.' It is formulated thus: The truth of the consequent follows from the truth of the antecedent, and the falsehood of the antecedent from the falsehood of the consequent. This law expresses the nature of the necessary relation affirmed to exist between them.

The Fallacies of the Hypothetical Syllogism. To attempt to draw a conclusion by denying the antecedent or by affirming the consequent gives rise to error. The reason for this is apparent so soon as we realize that one event may have more than one antecedent. If we have information that tells us that *B* occurs only when *A* occurs, we are able to build up four valid hypothetical syllogisms: —

> If *A*, then *B*; *A* is; therefore *B* is.
> If *A*, then *B*; *A* is not ; therefore *B* is not.
> If *A*, then *B*; *B* is ; therefore *A* is.
> If *A*, then *B*; *B* is not ; therefore *A* is not.

In such a case, we are able to move in both directions because *B's* existence is bound up only with *A's*. But ordinarily this is not the case. Hence *B* may exist even when *A* does not. An example may make this clearer: —

If the harbor is frozen, the ships cannot come in ;
The harbor is not frozen;

Can we draw a conclusion in this case? No; because there may be other reasons why the ships cannot come in. The harbor may be mined, or there may be a storm.

Reduction to Categorical Form. It has been customary to reduce hypothetical syllogisms to the categorical form and to find analogies therein for these two fallacies. But to reduce the simple to the complex is wasted effort, and there is little doubt that much of our thinking falls naturally into the hypothetical form. It would seem more reasonable to reduce categorical syllogisms to this simpler form. For the sake of completeness, I shall perform both reductions.

If war is not declared, our country will escape disaster;
But war will not be declared;
Therefore our country will escape disaster.

Reduced to categorical form, this becomes: —

All cases of war not being declared are cases of our country escaping disaster;
This is a case of war not being declared;
Therefore this is a case of our country escaping disaster.

It will be noted that a relation of events is changed into an inclusion of classes. As a matter of fact, the major premise of a categorical syllogism usually states a principle, and the minor premise an instance. Its meaning is often more clearly expressed as a hypothetical.

All men are mortal;
Socrates is a man;
Therefore Socrates is mortal.

really means: —

If man, then mortal;
Socrates is a man;
Therefore Socrates is mortal.

'All men are mortal' is not an enumerative proposition, but the statement of a relation between man's nature and his mortality. If it were enumerative, we should already know that Socrates was mortal. The logician must be the first to recognize that language forms are often accidental.

The Disjunctive Syllogism. In a disjunctive syllogism, the major premise is disjunctive, while the minor is categorical. Thus, the major premise expresses alternative possibilities among which the minor makes a selection. 'The inventor of the calculus was either Leibnitz or Newton,' and 'That bird was either a sand martin or a swallow,' are typical disjunctives. The minor premise either asserts or denies one of the alternatives. The conclusion either denies or asserts the other. For example: —

This man is either very clever or a cheat;
He is not very clever;
Therefore he is a cheat.

The source of danger is in the major premise. The alternatives there offered must be exclusive of each other. Unless this is the case, the affirmation of one of them does not enable us to make an assured statement. Hence, we may say that the ideal form of disjunction is that between contradictories. 'Sempronius is either honest or dishonest,' 'This man acquired his wealth worthily or by fraud,' 'He was either married or unmarried' are examples of clear-cut exclusion. On the other hand, a man may be both a knave and a fool. If

so, the assertion that he is a fool does not enable us to proclaim that he is not a knave. Again, it is only too common to assume that a man must be moved either by self-interest or by altruism, whereas motives are nearly always mixed. Great care must be taken in formulating the major premise of a disjunctive syllogism.

The Dilemma. A dilemma is technically defined as an argument in which the major premise is a complex hypothetical proposition and the minor is disjunctive. Practically, it means the presentation of two or more alternatives, all of which are unpleasant.

A dilemma may be constructive or destructive, simple or complex. If the antecedents of the hypothetical major are affirmed in the minor premise, the dilemma is said to be 'constructive'; if the consequents are denied, it is 'destructive.' Thus the rule at work is the Law of Reason and Consequent. The new element is the disjunctive minor. A dilemma is simple when the consequents are the same for both antecedents; complex, when the consequents of the hypothetical major are not the same for both antecedents.

The following is an example of a *simple constructive dilemma:* —

> If A is B, C is D; and if E is F, C is D;
> But either A is B or E is F;
> Therefore C is D.

If a man acts in accordance with his own judgment, he will be criticized; and if he is guided by the opinions and rules of others, he will be criticized;

But he must either act in accordance with his own judgment, or be guided by the opinions of others;

Therefore, in any case, he will be criticized.[1]

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[1] Creighton, *An Introductory Logic.*

The structure of the *complex constructive* dilemma is exemplified in this argument: —

> If a statesman who sees his former opinions to be wrong does not alter his course he is guilty of deceit; and if he does alter his course he is open to a charge of inconsistency;
> But either he does not alter his course or he does;
> Therefore, he is either guilty of deceit, or he is open to a charge of inconsistency.

It should be noted that the conclusion of this argument is disjunctive. The following is an oft-quoted complex destructive dilemma: —

> If this man were wise he would not speak irreverently of the Scripture in jest; and if he were good he would not do it in earnest;
> But he does it either in jest or in earnest;
> Therefore he is either not wise or not good.

There are three ways of meeting a dilemma, all of which have picturesque names. One may try to *escape between the horns* of a dilemma, these being the alternatives on which you are to be *impaled.* To escape is to show that there is some other alternative. One may try to *rebut* a dilemma by constructing another with a contradictory conclusion. Or, finally, one may try *to take a dilemma by the horns* by accepting an alternative and proving that the consequence asserted does not follow.

Since it is difficult to get an exhaustive disjunction for the minor premise, it is usually easiest to escape between the horns. The rebuttal of a dilemma is an interesting process, and many examples have come down to us from antiquity, chief of which is the story of Protagoras and Euathlus. This story, called *Litigiosus*, is as follows: —

Protagoras had agreed with Euathlus to teach him rhetoric for a fee, of which half was to be paid at the conclusion of the instruction, and the remainder when Euathlus won his first suit in court. Observing that the latter delayed to practice, Protagoras thought he was endeavoring to evade payment, and therefore himself brought a suit for the recovery of the second half of his fee. He then argued with the jury that Euathlus ought to pay him, in the following way:—

> If Euathlus loses this case, he ought to pay by the judgment of the court; and if he wins it, he ought to pay by his own agreement;
> But he must either lose it or win it;
> Therefore he ought to pay.

Euathlus rebutted this dilemma by a counter-dilemma:—

> If I win this case, I ought not to pay by the judgment of the court; and if I lose it, I ought not to pay by my own agreement;
> But I must either win or lose it;
> Therefore I ought not to pay.

An examination of these two dilemmas shows that each antecedent has two consequents, one only of which is used in each dilemma. There are, in other words, two standards and each disputant uses them only when convenient. This conflict of standards must first be settled before the problem can be cleared. Has Protagoras ground for action? He could sue him only for refusing to plead. But is there evidence that Euathlus had agreed to become a pleader?

The dilemma is a controversial instrument. When validly constructed, it is very effective. But it is an instrument which may be turned against the user and

then becomes a trap in which the hunter is himself taken. Before resorting to the dilemma, one should be certain that the situation is itself dilemmatic.

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CHAPTER XIV

FALLACIES IN ARGUMENTATION

What a Fallacy is. A fallacy is, broadly speaking, an error in reasoning. This error may occur at any step. We may misinterpret our perceptions, or classify things wrongly, or work out bad definitions, or confuse ideas, or draw invalid conclusions from premises. The risk of misjudgment is constantly present. The task confronting the human mind is twofold, that of attaining knowledge and that of handling it properly after it has been attained; and neither part of this task is easy. Mistakes are constantly being made. But since human beings must think, because of the very requirements of their life, the best they can do is to examine every stage and aspect of thinking in order to reduce mistakes to a minimum. It was this need which gave rise to logic. We may say, then, that thinking is an adventure and that fallacies are misadventures.

A Classification of Fallacies. The traditional classification of fallacies is into 'deductive' and 'inductive' fallacies. We shall interpret this classification as corresponding to the twofold task referred to above. Man must correctly handle the knowledge he already has, both in his private thinking and in his arguments with other people; and he must be able to make investigations into new fields. Where errors creep into one's own thinking or into argumentation, although no essentially new data or principles are involved, we have

to do with 'deductive' fallacies or fallacies in argumentation; where, on the other hand, errors arise in attempts to extend the boundaries of knowledge, they are better called 'inductive' fallacies. In the present chapter, we are concerned primarily with the former.

Classification of Deductive Fallacies. It is difficult to get a single positive principle of classification for fallacies. The usual division is into 'formal' and 'material.' Formal fallacies are those which involve some breach of the logical rules of the syllogism. Since obversion and conversion are closely connected with the syllogism, false conversion and false obversion may be placed here. Material fallacies result from a misapprehension of the content or from false assumption.

Formal fallacies have been treated in detail in connection with the chapters devoted to the syllogism. I shall run over the list and suggest that the student seek to clear up his memory in regard to them. They are as follows: (1) *False conversion;* (2) *False obversion;* (3) *Four Terms;* (4) *Undistributed Middle;* (5) *Illicit Major;* (6) *Illicit Minor;* (7) *Two Negative Premises;* (8) *Two Particular Premises;* (9) *Affirmative Conclusion with Negative Premise;* (10) *Denying the Antecedent;* (11) *Affirming the Consequent;* (12) *Incomplete Disjunction.* Of these, special stress should be put on the Undistributed Middle and the fallacy of Four Terms. All verbal fallacies appear syllogistically as Four Terms. Whenever the middle term is vague or indefinite, there is danger of an ambiguous middle. While M is P, yet S may be *M with a difference.*

Material fallacies fall into two main divisions, those

connected with definition and those involving unwarranted assumption. If terms are not clearly defined, some form of ambiguity is almost sure to appear. On the other hand, the logic of argumentation demands that the conclusion be not assumed but deduced from accepted premises. The following classification will serve as a guide: —

Material Fallacies

Fallacies of Equivocation	*Fallacies of Unwarranted Assumption*
1. Ambiguous and Shifting Terms.	1. Begging the Question.
2. Amphiboly.	2. Complex Question.
3. Composition.	3. Irrational Evidence.
4. Division.	(*a*) *Argumentum ad hominem.*
5. Accident.	(*b*) *Argumentum ad populum.*
6. Accent.	(*c*) *Argumentum ad ignorantiam.*
7. Figure of Speech.	(*d*) *Argumentum ad vericundiam.*
	4. Irrelevant Conclusion.
	5. *Non sequitur.*

Fallacies of Equivocation. 1. Words may be indefinite and yet not ambiguous. When, however, there is the slightest danger of misunderstanding, words should be defined or else clearly qualified by the context. In literature, the suggestiveness of a word may depend upon its capacity to awaken many lines of association. But precision rather than suggestiveness is fundamental for reasoning. Especially is this the case in long arguments in which the meaning of a term may shift from stage to stage if care be not taken. For instance, an argument may start with legal right and conclude with moral right. The following fallacious

syllogism may serve as an example of a subtle form of equivocation called 'Ambiguous Middle':—

> All able men are consistent with themselves;
> He who changes his opinions is not consistent with himself;
> He who changes his opinions is not an able man.

A moment's reflection shows that the reference to time is different in the two premises.

2. When the ambiguity lies in the very structure of the sentence, the resultant fallacy is called 'Amphiboly.' The sentence permits of a double interpretation. Such cases as occur to-day are accidental and have little significance except as a warning against careless writing. The following want-advertisements are good examples:—

> Wanted: a groom to look after two horses of a pious turn of mind.[1]
> A second-hand morris-chair is wanted by a bachelor with richly carved claw-feet.

The classical cases of Amphiboly are the decrees of the oracle at Delphi. The response of the oracle to Pyrrhus is an excellent instance: "Pyrrhus the Romans can, I say, subdue."

3. The fallacy of Composition arises when an attribute is predicated of a whole, which holds only of a part. The object to which the property is fallaciously assigned is usually an aggregate. Thus, to argue that a country is prosperous because many business men are making money, is essentially to commit the fallacy of Composition. Very similar to this is the following argument:—

[1] Gibson, *The Problem of Logic.*

The manufacturers of woolens are benefited by the duty on woolen goods; the manufacturers of cotton by the duty on cotton; the farmer by the duties on wool and grain; and so on for all the other producing classes; therefore, if all the products of the country were protected by an import duty, all the producing classes would be benefited thereby.

It will be noted that the mind easily tends to pass from individual cases and classes to larger wholes without thought of the new influences and relations which are at work in such a whole.

4. The fallacy of Division is the converse of Composition. It consists in the assignment to the part of attributes which are true only of the whole. We can bring this fallacy into touch with the distinction between the collective and the distributive use of terms by saying that it arises from a disregard of these two uses. In the major premise, the term may be used collectively, while it is employed distributively in the minor. The following example illustrates this hasty movement from the whole to the parts: —

He cannot be innocent, for he was a member of the mob which committed the deed.

We are especially liable to fall into this error in discussions upon social and political topics. Because people act rather foolishly or selfishly as units in a group, it does not follow that their conduct will be of the same character when they are acting as individuals. As one writer has forcefully put it, individual morality has outstripped crowd morality by many centuries. "Perhaps the commonest form of the fallacy is that which it takes in such arguments as, 'It must be wrong for you to act in this manner, because if every

one did so, the consequences would be disastrous.' We start by urging that if A *and* B *and* C . . . (conjunctive) acted in some specified manner — the welfare of the world would be fatally affected, and we go on to argue that no less fatal consequences must follow when A *or* B *or* C (disjunctive) act in the manner specified."[1]

5. The fallacy of Accident consists in the application of an abstract principle without allowance for qualifying conditions. Because water boils at 212° Fahrenheit and such water will boil an egg in five minutes, it does not follow that boiling water *on the top of a mountain* will cook an egg in that number of minutes. Because charity is a virtue, we must not conclude that it is necessarily virtuous to give a beggar a dollar. It may do him more harm than good. Such abstract principles are not true universals and the mistake lies in so taking them. In argument, it is so easy to make general pronouncements and shove to one side those qualifications which a keener desire for truth would take account of. "There is no fallacy more insidious than that of treating a statement which for many purposes is true as if it were true always and without qualification."[2]

The converse fallacy of Accident consists in the passage from what is true under certain circumstances to an abstract principle based upon these instances. Inductively, it takes the form of a hasty generalization. An individual who argued that a college education was

[1] Gibson, *The Problem of Logic*, p. 284. Cf. Welton, *Manual of Logic*, vol. II, p. 247.

[2] Joseph, *An Introduction to Logic*, p. 549.

valueless because Edison and Ford did not have one would be committing this fallacy. The type of argument is, I think, easily recognized.

Both the direct and the converse fallacies of Accident are important because they bear witness to a deep-seated tendency to think abstractly and dogmatically. Only the careful thinker does justice to both principles and facts. Lotze, a German philosopher of the last century, wrote the following very sane comment: "Two general modes of fallacious thought are developed by the habitual commission of these fallacies, and illustrate them on a grand scale. The first is doctrinairism, the second narrow-mindedness. The doctrinaire is an idealist who refuses to see that though ideas may be right in the abstract, yet the nature of the circumstances under which and of the objects to which they are to be applied, must limit not only their practicality, but even their binding force. The narrow-minded, on the other hand, can recognize and esteem no truth and no ideal, even the most universally valid, except in that special form to which they have become accustomed within a limited circle of thought and personal observation. Life is a school which corrects these habits of mind. The parochially minded man sees things persist, in spite of himself, in taking shapes which he considers unprecedented, but he finds the world somehow survives it, and learns at last that a system of life may be excellent and precious, but that it is rash from that to argue that it is the only proper mode of orderly existence. And the enthusiast for ideals, when he sees the curtailment which every attempt at realization inflicts on them, learns the lesson

which the disjunctive theorem might have taught him."[1]

6. The fallacy of Accent is due to a misleading emphasis. When words are taken from their context, they often seem to have a meaning which an investigation shows was not the intent of the author. This error is near enough to misinterpretation caused by misplaced emphasis to be classified with it. Another instance is the use of italics in a quotation where no words were originally italicized. Jevons points out a laughable case of the use of italics lending itself to misunderstanding. In the First Book of Kings, XIII, 27, the translators added a word to complete the sentence and put it in italics to indicate this fact. The result was as follows: "And he spake to his sons, saying, Saddle me an ass. And they saddled *him*." "It is curious," continues Jevons, "to observe how many and various may be the meanings attributable to the same sentence according as emphasis is thrown on one word or another. Thus the sentence, 'The study of logic is not supposed to communicate the knowledge of many useful facts,' may be made to imply that the study of logic *does* communicate such a knowledge, although it is not supposed to do so; or that it communicates a knowledge of many *useless* facts."[2]

7. The fallacy of Figure of Speech is due to the ambiguous character of some verbal inflection. It is not of much importance in a language like English in which inflection is at a minimum. The best case, and the one usually quoted by logicians, is found in Mill's book, *Utilitarianism*. He is trying to prove that pleasure is

[1] *Logic*, vol. II, p. 5 (Eng. trans.).

[2] *Lessons in Logic*, p. 175.

the chief good. "The only proof," he writes, "capable of being given that an object is visible is that people actually see it. The only proof that a sound is audible is that people hear it: and so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable, is that people actually desire it." But 'visible' and 'audible' mean what *can* be seen and heard, while 'desirable' means what is worth desire or *ought to be* desired. Mill did not notice the change in meaning because he was paying too strict attention to the form of the words. A similar mistake is sometimes made when the original or etymological meaning of a word is stressed and the more usual meaning of the present disregarded. Thus, to argue that representatives must *passively represent* their constituents is to commit this fallacy.

Fallacies of Unwarranted Assumption. These fallacies consist in the entrance into the argument of an unwarranted element of assumption. Either the point at issue is assumed (Begging the Question, and Complex Question), or the evidence adduced is irrelevant (Irrational Evidence), or the conclusion is beside the point (Irrelevant Conclusion, and *Non Sequitur*). In all these cases, the conclusion is not demonstrated and the fallacy consists in the assumption that it is.

1. *Begging the Question*, or *Petitio Principii*, is the assumption of the point to be proved. In an argument, it consists in the taking for granted what an opponent would not admit if its significance were understood; it is the "surreptitious assumption of a truth you are pretending to prove."

There are two distinguishable types of question-

begging. In the one, the conclusion we wish to prove is directly assumed under another name; in the other, some larger general principle which includes the point in dispute is taken for granted.

The first type appears either as a *question-begging epithet* or as *reasoning in a circle.* To call a prisoner before the bar a worthless good-for-nothing whom every one has suspected for a long time is not to prove that he has committed the crime of which he is accused, but to prejudice the jury against him. Epithets are insidious attempts to lead the minds of the listeners in the desired direction independently of genuine evidence. Nowhere is this procedure more frequently resorted to than in politics. Mud-slinging is the bane of political life, and, unfortunately, it has its bad ethical effect. Those who drop into the practice must lose some measure of their intellectual integrity. Logic has its moralities. Reasoning in a circle consists in the use of a premise which is only the desired conclusion stated in other words. It is difficult to offer a good example in brief compass of this species of Begging the Question, since it is more apt to occur in a long argument. Often arguments do not make any advance, but twist and turn and repeat the same things in different words. Absence of a clear-cut argument is nearly always a sign of an element of circular thinking. The individual tries desperately to make an advance, but really turns around the same fixed ideas with much noise of asseveration. It is a form of logical hemming and hawing. When there is a sophistic element, an intent to deceive and to secure victory at all hazards, reasoning in a circle becomes argument in a circle. A frequently used device in Eng-

lish is to state the proposition to be proved in words of Saxon origin and then to give as reason for it the same proposition in words of classical origin. The following example from Whately is much quoted in this connection: "To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the State, for it is highly conducive to the interests of the community that each individual should enjoy a liberty perfectly unlimited of expressing his sentiments."

The second general type of question-begging consists, as we have said, in the assumption of a larger principle than the one in dispute from which this latter may be deduced. It has much in common with the fallacy of Accident. Thus, if a piece of legislation concerning a particular remediable wrong were under discussion, it would be question-begging to make appeal to some abstract principle, like 'All legislation which interferes with the right of free contract is bad.' At one time — in the past, I am thankful to say — certain abstract principles of political economy were used in this way as maxims against remedial legislation; and we all know how both the conservative and the demagogue make use of slogans.

In regard to this fallacy, it is well to bear in mind what De Morgan called its opponent fallacy: "It is the habit of many to treat an advanced proposition as a begging of the question the moment they see that, if established, it would establish the question." Such an attitude is not playing fair.

2. When a question is so stated as to involve a questionable assumption, it is called a 'Complex Question.'

Any direct answer to such a question implies the admission of the point assumed. Complex Question may be a trick as in the following examples: 'Have you left off beating your wife?' 'Has the practice of heavy drinking ceased in your part of the country?' Or it may be due to the fact that the interrogator has not analyzed all that his question involves. A teacher in philosophy may be asked how he explains telepathy.

3. "The characteristic appeal which Science, in its processes of reasoning, makes to the mind is the *argumentum ad judicium*, or appeal to reason. When the appeal is *not* to the impartial reason, but to the feelings, passions, prejudices of men, it is, from the logical point of view, radically *irrelevant*, and involves the fallacy of Irrational Evidence."[1]

This fallacy is akin to the first type of Begging the Question; the difference lies in the openness of the appeal to the non-logical. Locke's treatment of the various forms of this fallacy is classic. "Of all the arguments," he writes, "that men ordinarily make use of, the argument *ad judicium* alone brings true instruction with it, and advances us in the way to knowledge. . . . For (1) it argues not another man's opinion to be right, because I, out of respect, or any other consideration but that of conviction, will not contradict him. (2) It proves not another man to be in the right way, nor that I ought to take the same with him, because I know not a better. (3) Nor does it follow that another man is in the right way because he has shown me that I am in the wrong."[2] Putting these conclusions into touch with the experience

[1] Gibson, *The Problem of Logic*, p. 287.

[2] Locke, *Essay concerning Human Understanding*, bk. IV, chap. XVII.

that any appeal to feeling must be regarded with suspicion, logicians have distinguished the following varieties of Irrational Evidence: —

(*a*) *Argumentum ad hominem.* In this fallacy the argument is directed against the character of the man who is the opponent instead of adhering to its proper task of proving the point at issue. Too often, calumny and gossip are resorted to in order to make out a case and win the decision. Here, again, logic touches upon ethics. If the ideal held is that the end justifies the means, such arguments come under rhetoric and the only point of attack is their effectiveness. They are more non-logical than illogical. As a matter of fact, they are usually employed by individuals who are fully aware of what they are doing. They are the tricks of the sophist. A story is told of O'Connell that on one occasion, when he had to defend a man who was clearly in the wrong, the counsel for the prosecution was a certain Mr. Kiefe, who had come in for some money in rather a questionable way, and had taken the name of O'Kiefe. O'Connell commenced his defense by addressing his opponent: —

> "Mr. Kiefe O'Kiefe,
> I see by your brief o'brief
> That you are a thief o'thief," —

which so disconcerted Mr. O'Kiefe and so tickled the jury that a verdict was returned for the defendant. At least there was wit in this case of *argumentum ad hominem.* Any argument in which there is appeal to motives and facts which do not bear upon the objective truth of the position at issue can be classed under this title.

(*b*) *Argumentum ad populum* is an appeal to the

feelings, passions, and prejudices of the group addressed instead of to the intellect. Almost all political speeches and, I fear, nearly all writings which have become popular in times of excitement, have contained an ingredient of emotional incitement. The logician must follow the psychologist in the study of crowd psychology. He will then better know the power of suggestion. The individual thinks most clearly and independently when he is alone. Let him be on his guard against suggestions of an emotional sort which do not leave a deposit of definable ideas. The thinker needs the will to analyze and the power to put passion and prejudice far away from him. Let him remember that we call 'enthusiasm' in ourselves and those who think like us what we call 'fanaticism' in our opponents. The ideal of objectivity is not a bad thing to set before one.

The best instance in literature of *argumentum ad populum*, according to one writer,[1] is Anthony's speech to the Roman mob in Shakespeare's *Julius Cæsar*. Let the student pick up his high-school copy and re-read the passage.

A species of *argumentum ad populum* sometimes referred to is *argumentum ad misericordiam*, an appeal to pity or sympathy for a cause or person when the facts do not warrant it. We Americans are soft-hearted and sometimes foolishly sentimental.

(*c*) *Argumentum ad ignorantiam* consists in the attempt to throw the burden of disproof upon the other party to an argument. To maintain that telepathy is true just because it cannot be positively disproved is an example of such a policy. Mystics are constantly

[1] Taylor, *Elementary Logic*, p. 180.

resorting to this fallacy. As Creighton points out, the reasoning seems to be as follows: —

> It is not impossible that this is so;
> What is not impossible is possible;
> Therefore it is possible that this is so;

Mere abstract possibility is not enough; there must be some evidence of a factual character.

(*d*) *Argumentum ad vericundiam* is an appeal to a man's modesty in the face of the reverence people feel for authority and tradition. It is fallacious because it involves the refusal to examine a case on its real merits and the willingness to give more weight to the *ipse dixit* of accepted authorities than to reason and fact. This fallacy is rooted in the conservative and conformist instincts of society. Locke felt the weight of the scholasticism of his day and his quaintly phrased protest is well worth quoting: "When men are established in any kind of dignity, it is thought a breach of modesty for others to derogate any way from it, and question the authority of men who are in possession of it. This is apt to be censured, as carrying with it too much pride, when a man does not readily yield to the determination of approved authors, which is wont to be received with respect and submission by others." We must give weight to the views of men with established reputations, but there is no obligation to be too docile.

Another form of the same fallacy is the tendency to transfer reputation from one field to another. To quote a famous inventor on a point of theology, an energetic business man on problems of broad statesmanship, or an explorer on questions of social ethics is to commit this fallacy. A man may be competent in one field and

have no special competence in another. The logician must try to hold before the mind the meaning and conditions of competence. To tell what constitutes proof is also to tell what is *not* proof.

4. *Irrelevant Conclusion* consists in proving something different from the actual point at issue. It is an evasion or ignoring of the proposition to be proved. "Instead of proving that 'this prisoner has committed an atrocious fraud,' you prove that the fraud he is accused of is atrocious: instead of proving (as in the well-known tale of Cyrus and the two coats) that the taller boy had a right to force the other boy to exchange coats with him, you prove that the exchange would have been advantageous to both: instead of proving that the poor ought to be relieved in this way rather than in that, you prove that the poor ought to be relieved." [1]

This fallacy is most easily committed in a long argument in which the original question passes from the mind in the heat of the debate. One point leads to another, and, if some one of the contestants is not clear-headed, the concluding thesis is apt to be decidedly different from the beginning one. "The person who commits the fallacy of irrelevancy is, of course, quite unaware of his error, unless his purpose be to mislead. He does not see that the proposition which he proves is related to the point at issue only through a questionable assumption. But in criticizing an irrelevant argument it is not necessary to point out the nature of the assumption which underlies the argument. Merely to show that what is proved is not what ought to be proved or what is supposed to be proved, answers the purpose of criti-

[1] Whately, *Elements of Logic.*

cism. 'True, but irrelevant,' is often the most concise and effectual criticism. 'Thus, when in a discussion one party vindicates, on the ground of general expediency, a particular instance of resistance to government in a case of intolerable oppression, the opponent may gravely maintain that we ought not to do evil that good may come — a proposition which, of course, had never been denied, the point in dispute being whether resistance in this particular case were doing evil or not. Or again, by way of disproving the assertion of the right of private judgment in religion, one may hear a grave argument to prove that it is impossible every one can be right in his judgment.' The first of these arguments assumes, as a second premise, that 'this is a case of doing evil that good may come'; while the second takes for granted that 'if every one cannot be right in his judgment, then private judgment should not be permitted.' These assumptions, however, are highly questionable, and until they are proved the arguments are beside the point."[1]

All who have argued much with people are aware of how difficult it is to keep their attention fixed on the point which first arose for discussion. This tendency to wander off the track is the psychological cause of such irrelevancies as do not have a sophistic origin.

5. *Non sequitur* is the name usually now given to a conclusion which does not follow from the premises. The premises may be quite true, but they do not lead to the proposition which is attached beneath them. In the case of this fallacy, as in practically all of those which come under the caption, 'Unwarranted Assump-

[1] Bode, *An Outline of Logic*, pp. 103–04.

tion,' the argument in which it occurs is nearly always long-winded. If we are not very much interested in the assumption, or if we are not inclined to be analytic, a boldly advanced conclusion may be accepted at its face value. The following illustration of De Morgan's is a fair sample:—

Episcopacy is of Scripture origin.

The Church of England is the only Episcopal Church in England.

Therefore, the church established is the church that should be supported.

I am willing to hazard the opinion, that such an argument, extended over many pages by means of historical details, would convince those who were already favorably inclined.

Conclusion. The chief advantage in a summary discussion of the fallacies lies in the training of the attention it involves. It is a well-known principle of psychology that we can see best what we are looking for. Now, when we feel that an argument is fallacious, it is of great assistance to have in mind certain general types of fallacy which are possibly relevant. Once the classification is made, a standard interpretation and description of the fault is within our grasp.

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CHAPTER XV

THE NATURE OF INDUCTION

Induction and Deduction. Very often in the history of a science, inappropriate terms fasten themselves on the terminology and are with difficulty removed. Logic presents us with such a case which is of the nature of a false contrast. Until very recently, it has been customary to divide logic into two distinct parts called, respectively, 'deductive' and 'inductive' logic. But it is being ever more clearly realized that this division is not satisfactory. What has been called 'inductive logic' is really the logic of scientific method or the logic of all continuous and systematic investigation in a comparatively new field where principles are still to be discovered. *And the logic of systematic investigation deals with the whole of concrete reasoning; it concerns itself with the discovery of facts, their classification, correlation, and explanation.*

If logic is the science of correct reasoning, it must examine the principles and methods used in such systematic investigation. It must study the nature of systematic inference from facts and the construction of systems of knowledge as well as the more formal principles of classification, definition, and syllogistic argumentation. Logicians have realized this necessity and have given increasing attention to the larger field thus opened. As a consequence, every recent text devotes much of its space to scientific method and to the dangers which confront the various stages of concrete inference.

Now, when we examine concrete reasoning which does not take its facts and principles for granted, we discover that it includes both induction and deduction. These are really supplementary and inseparable phases of any complete act of reasoning. The mind passes back and forth between fact and theory, evidence and inference, and, in so doing, both elements are modified. In other words, principles and hypotheses are always tentative and experimental, while facts need selection and interpretation.

The term 'induction' is often used roughly for the passage from facts and less general propositions to laws and more general propositions; while 'deduction' refers to the process of analyzing ideas and using them in the interpretation of facts. As we proceed, we shall realize ever more clearly that the solution of any problem involves the continuance of this double movement in the course of which facts are gathered, selected, and interpreted, and principles are suggested and developed. Induction is, then, only a *phase* of concrete reasoning or investigation; it is not a process which can exist apart. But because we have neglected this phase, relatively, up to now and have laid stress upon language, definition, and the syllogism, we must emphasize it in the following chapters.

A Glance at the History of Logic. What we have wished to point out is that the terms, 'deductive' and 'inductive,' were applied to the older and newer phases of logic as the result of an historical accident and that they are misnomers. The older logic tended to be static and to stress consistency and implication, while the newer developments were filled with the spirit

of science and stressed its methods and mode of investigation; the older logic was one of consistency and order, the newer was one which concerned itself with the laws of systematic and progressive inference. Now, the syllogism, which shows how a conclusion follows inevitably from the subsumption of an instance under a rule, was the dominant element in the older logic and this was therefore called 'deductive'; the movement from fact to law was the conspicuous feature of the newer developments and the resultant logical doctrines were classified together as 'inductive.' Thus these terms stood for an historical contrast.

But this historical contrast has lost its sharpness with the lapse of time. There is no longer need to thunder against the scholastics as did Locke and Francis Bacon. Logic is, after all, one science and not two. The old factions have disappeared, and thinkers on logic see clearly that consistency and growth are not opposed to each other. The older logic has consented to be modified by the spirit of the new. Logic has become less formal all along the line, and lays more stress upon the material of thought and the importance of purpose.

Since we have not taken much space for the history of our subject, a brief reference to the purposes dominating logic at different epochs may be advisable. "Perhaps the simplest way of disentangling the leading features of the departments of Logic is to take them in relation to historical circumstances. These features are writ large, as it were, in history. If we recognize that all bodies of doctrine have their origin in practical needs, we may conceive different ages as controlled each by a distinctive spirit, which issues its mandate

to the men of the age, assigning to them their distinctive work. The mandate issued to the age of Plato and Aristotle was, *Bring your beliefs into harmony one with another*. The Aristotelian logic was framed in response to this order: its main aim was to devise instruments for making clear the coherence, the concatenation, the mutual implication of current beliefs. The mandate of the Mediæval Spirit was, *Bring your beliefs into harmony with dogma*. The mediæval logic was contracted from Aristotle's under this impulse. Then, as science developed, a new spirit was roused the mandate of which was, *Bring your beliefs into harmony with facts*. It was under this impulse that a body of methodical doctrine vaguely called Induction gradually originated."[1]

Now, the first and last mandates, at least, are permanent ones which will always correspond to a need of human life. Must we not always try to bring our beliefs into harmony with one another and with fact? In exposition there is a good pedagogical reason for treating the logic of consistency first; it is such a systematic body of doctrine pivoting around the syllogism. But in the logician's thought it must be brought into touch with concrete reasoning in order to gain in vitality. The student, also, must be prepared to make this synthesis.

The Function of the Syllogism. The chief aim of logic for many centuries was, then, to determine the conditions of correct conclusion from accepted premises. Because of this limited aim the syllogism was the pivot around which logical analysis turned. This simplification of logic had its advantages, but likewise its disad-

[1] Minto, *Logic, Inductive and Deductive*, pp. 243–44.

vantages. Because the syllogism was not seen in its larger setting as only a part of actual reasoning, it was formalized and mechanized. The chief difficulties of creative thinking were ignored. While the keener minds were aware that it gave only a logic of consistency and order, the majority deluded themselves into assuming that it was an instrument of truth and of genuine inference. This misinterpretation was, however, due largely to the time-spirit of dogmatism which prevailed for so long a period. With the advent of science, logic went into the melting-pot and is only gradually re-forming in the mould of a wider outlook and a truer perspective. The syllogism still remains intact, but a juster view of its function is held. It is now seen that its function and even its character depend upon the *purpose* which is uppermost.

When the purpose is the limited one of testing the consistency of a given argument, the syllogism is a valuable instrument. It offers a technique by which the argument can be analyzed and its parts seen in their mutual relations. We can decide whether all the elements of a complete argument are present, whether the terms are really only three in number, whether the middle term has the same meaning in the two premises, whether the conclusion follows, etc. In short, the syllogism and its theory presents the would-be analyst with satisfactory methods for his task. He knows what to demand and where the dangers lie. Relatively to this purpose, the syllogism can no more be outgrown than human thought itself.

But this purpose is, after all, a narrow one. We are more frequently interested to-day in the increase of

knowledge and in its factual verification. We want to see how knowledge grows through investigation and the formation of hypotheses. We are more inclined to go back of the premises of an argument and to consider principles for which evidence is not advanced as dogmas. Moreover, probability plays a larger part to-day than ever before, and it will be remembered that the syllogism is suited only to certainty. Hence argument is not so easy a matter as it once was. Not only must an argument be self-consistent; it must also be consistent with the facts. But purposes can coexist; the narrower ideal of formal consistency has its validity just as has the wider purpose. The wider purpose must be able to include the narrower.

When the syllogism is a part of inductive-deductive reasoning, the premises are essentally tentative: they lead to a certain conclusion; but if this conclusion does not agree with the facts, the premises must be modified. The syllogism is a phase of a larger process. We shall understand this aspect of the case better when we come to consider the part played by hypotheses in modern science. It is important to us at present chiefly as showing how the older doctrines are being modified by a larger setting.

Why the Syllogism is only a Part of Reasoning. In the formal doctrine of the syllogism, the most difficult problems of actual thinking are disregarded. If your opponent is kind enough to grant you certain premises, he is undoubtedly obliged to admit the truth of the implied conclusion. But whence do these premises come? And is your opponent obliged to admit their truth? If he does not, the argument is at a

standstill until premises are agreed upon. Otherwise, the syllogism involves a begging of the question. It is obvious that the syllogism is a part of a larger whole unless appeal to intuition is permitted.

Let us verify this conclusion by an examination of the three kinds of syllogism. The categorical syllogism deals with classes, but it does not tell us how classes are formed and tested. Yet such formation and testing constitute one of the most difficult tasks of science. If we bear in mind the fact that real argument arises only as the result of doubt and perplexity, we can appreciate the importance of the processes largely preliminary to the syllogism. All syllogisms require at least one universal proposition, but universal propositions do not grow on every bush.

The hypothetical syllogism is in essentially the same situation. The major premise states a universal or necessary connection between an antecedent and a consequent. But such principles must first be achieved, and they represent the maturity of a science rather than its beginning. Otherwise, they are essentially experimental. So long as the major is tentative, the conclusion is a challenge to fact; but then the syllogism becomes a part of a progressive movement of knowledge.

The disjunctive syllogism is obviously in the same situation. The alternatives must be complete and mutually exclusive. But the attainment of such alternatives in any new field is not an easy matter. Of course, it is an easy matter to work up such alternatives as honest and dishonest, truthful and untruthful. These are simple contradictories and are not apt to lead to confusion. But to prove that certain theories alone

hold the field is an achievement lying back of the disjunctive syllogism.

Steps in Systematic Investigation. There are at least *four stages* in any systematic investigation. These have been stated variously, and certain writers have felt themselves compelled to break up one or more of the stages into parts. I must warn the student against supposing that these stages are temporally distinct so that one stage can be completed before the next begins. The mind of the true investigator keeps them as contemporaneous as possible, and passes up and down his chain of reasoning from fact to theory and from theory to fact. In a sense, the active thinker observes while he theorizes and theorizes while he observes.

The first stage in systematic investigation is observation; the second is the formation of hypotheses; the third is the development of these hypotheses; and the fourth is their progressive testing and verification by renewed observation. These steps are fairly separable for logical analysis though they do not exist in such isolation in actual investigation. Each stage is confronted by characteristic dangers and involves logical and psychological principles of importance. Only by means of this massive and systematic advance with its constant hazards is new knowledge obtained.

Three Elements in Investigation. There are three elements or distinguishable mental processes in all investigation. We may call the first the inductive element *par excellence*. In both observation, the beginning of investigation, and verification, its end, the stress is laid upon facts. The essential principle of

this element in the attainment of knowledge is *fidelity to relevant fact.* Science has always stressed fact as the beginning and only firm foundation of knowledge.

The second element is the formation of hypotheses. The creative investigator must be capable of making fruitful conjectures as to the relations which hold between facts or lie back of them. He must have what one psychologist speaks of as a creative imagination. Francis Darwin in his biography of his father remarks as follows: "He often said that no one could be a good observer unless he was an active theorizer. This brings me back to what I said about his instinct for arresting exceptions; it was as though he were charged with theorizing power ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into importance." We shall have much to say of this constructive, conjectural element as we proceed.

The third element is the deductive reasoning out of the implications of hypotheses and theories to see what they imply in the light of knowledge already possessed. Some writers in logic speak of this element as 'deductive inference'; others speak of it as 'reasoning.' In any case, the significance of the hypothesis is developed as completely as possible and its implications noted. These implications lead back to the sphere of fact again and so we return to the inductive element.

The Problem of Generalization. As has been well said, syllogism never generalizes. How, then, do we secure universal or semi-universal propositions to serve as premises? The difficulty arises from the fact that

experience gives us only particular events and things. We note that this medicine seems to have a beneficial effect, and that this piece of metal has certain properties; but how do we go beyond the instance to a rule? We must, first of all, be careful to set the problem correctly. Modern psychology informs us that every instance is interpreted in the light of past experience at the time it enters consciousness. Things and events are noted as members of a familiar class or as members of a potential class. Observation and generalization, perception and interpretation, sensation and association go hand in hand. We interpret as we perceive and generalize as we observe. The mind is, moreover, selective and stresses what is regarded as important or even essential. If this medicine cures an ill, then curing this kind of ill is an essential property of this kind of medicine. We expect classes and other instances. Why? Because it is the very nature of the mind to note similarities and differences and so to classify. "It is for the sake of generalization that we observe at all, and the very act of observing intelligently is nothing else than the act of generalizing from what we observe." We do not so much pass from particular instance to particular instance as from a beginning class to a growing class.

A little reflection must convince us that the mental process leading to generalization does not depend upon the application of any consciously held principle — even though its logical validity may imply some such principle. As writers since the days of David Hume have pointed out, the child unconsciously generalizes things and events and builds up practical rules to act upon. Wild plums are very sweet when they are ripe; fish

bite best on cloudy days; the river will be swollen after a rain; a circus always has a band, etc. Such classifications are gradually built up because the process of generalization continually repeats itself in our minds; and our lives are guided at every moment by the maxims and rules thus obtained. What science does is to extend experience and to control it by completer analysis and experiment. It does reflectively and methodically what common sense has been doing all along.

This tendency to put two and two together, to regard one thing as an indication of another, is evidently a part of our mental constitution. This linkage of terms combined with the recognition of likenesses and differences accounts for a large share of what we call reasoning. Professor Dewey goes so far as to assert that the establishment of one thing as a sign of another is the *central factor of thinking*. "Reflection thus implies that something is believed in (or disbelieved in), not on its own direct account, but through something else which stands as witness, evidence, proof, voucher, warrant; that is, as *ground of belief*. At one time, rain is actually felt or directly experienced; at another time, we infer that it has rained from the looks of the grass and trees, or that it is going to rain because of the condition of the air or the state of the barometer. At one time, we see a man (or suppose we do) without any intermediary fact; at another time, we are not quite sure what we see, and hunt for accompanying facts that will serve as signs, indications, tokens of what is to be believed."[1]

The Implication of Generalization. Logic does

[1] Dewey, *How We Think*, p. 8.

not impugn the act of generalization, but tries to understand what takes place and what principle is implied. It is evident that an event acquires meaning as a sign of another event so that an *internal* relation between them is developed in the mind. It is also apparent that any one event is regarded as typical of a class. We pass quickly from *this event* to *an event of this class* as soon as our minds are preparing to generalize. Suppose a savage goes out on a hunt for game, and, as he steps into the forest, a rabbit crosses his path. It happens that he has no luck that day. If he is superstitious, he is liable to connect this lack of success with the appearance of the rabbit. But he has no reason to suppose that there is anything peculiar about this particular rabbit; hence it sinks back into the class. It is the rabbit as a recognizable class of animals that presages bad luck. Unless there is strong reason to the contrary, the individual instance tends to be merged in the idea of the class. Psychologically, the class is as primitive as the individual.

But if the logician does not impugn the tendency to generalize, he is aware, nevertheless, that a principle is involved which must be valid if the generalization is valid. This principle or postulate is usually called the 'Principle of the Uniformity of Nature.' *We assume, consciously or unconsciously, that there are universal connections or relations in nature.* This assumption, when formulated explicitly, is suggested by the way the mind works and the degree to which nature seems to recognize the tendency to relate terms as invariable signs of one another. But logicians have come to the conclusion that the principle cannot be proved by expe-

rience, but can only be relatively confirmed by it. For this reason, it is frequently called a 'postulate.'

How Generalization differs from Expectation. Animals develop expectations in much the same way that human beings do. A doctor's horse expects to turn in at a certain farmhouse because it has done so on previous trips. A dog anticipates a walk when his master takes down his hat from the wall. The formation of associations of this character is natural to the mind at fairly low levels as well as at the human level. Is generalization more than this? Though based on it in part, it certainly goes beyond mere expectation. Other mental processes than those of association enter in. The animal glides from the one event to the other without holding them both before the mind as distinct objects of attention. But man does hold both terms of the relation before his mind at one and the same time, and makes them and their internal connection a single complex object of thought. This the animal cannot do. We may say, then, that generalization is a higher development than expectation and depends upon the capacity to conceive two events as classes and to relate them internally as somehow bound together. A generalization is a more or less explicit assertion that two terms are related.

The Importance of Generalization. It may not be amiss to call attention once more to the importance of generalization. Were we literally confined to particulars, we should be unable to reason. The very nature of reasoning is the passage from what is given to what is not directly given. As we say, we *infer* that so-and-so will happen because something else has oc-

curred. This something else is taken as a sign of the event inferred. Thus we apply rules to our present experience in order to interpret it and to pass in conception beyond it. But such rules can come only from generalization.

Testing Generalizations. While we cannot prove the Principle of the Uniformity of Nature, we can test specific inductive inferences to see whether they really come under the principle. If they stand all the empirical tests we can apply, we call them laws of nature. Science has for centuries been engaged in the discovery of laws of this character, laws which the hasty observations and inadequate methods of common sense cannot discover. The generalizations of common sense are rules and maxims which hold *in the main* rather than laws which are universal or do not admit of exceptions. Why common sense is not able to formulate many universal propositions will become clearer to us as we proceed. Its maxims are not founded on analytic methods which exclude the irrelevant. The strength of science rests on the development of such methods and the technique which accompanies them. It is to the consideration of this technique and these methods in their general aspects that we now turn our attention.

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CHAPTER XVI

THE GENERAL METHODS OF SCIENCE

How Science developed. Science grew to its present stature by the efforts of men who attempted to solve problems which attracted their attention. There was much groping before suitable methods and a fairly satisfactory technique were worked out. The simplest aspects of nature were attacked before it was even possible to approach the more complex fields. The growth of science was the result of the painstaking care and genius of such men as Galileo, Kepler, Newton, Lavoisier, Liebig, and others too numerous to mention. Their sagacity, fertility of suggestion, experimental ingenuity, and thoroughness led to the gradual accumulation of facts and the appearance of just notions about the constitution and mode of working of the inorganic realm. Bit by bit, problems were properly defined, investigated, theorized over, experimented upon and explained. The technique of measurement was evolved, mathematics applied, and instruments invented. Thus, step by step, facts were discovered and fundamental principles like the 'laws of Kepler' and the theory of gravitation of Newton were enunciated. All this was a growth of the most subtle kind which substituted a world of laws and conceptual elements for the familiar mass of changing things. The early scientists were confronted by a world of happenings and of unanalyzed things, and it was with the greatest difficulty

that they learned how to handle it and make it tell its secrets. Let the student try to remove from his thought of the world such ideas as mass, inertia, gravitation, the laws of motion, the chemical elements, energy, etc., and he will better understand how much he owes to the constructive genius of these men.

Since our purpose is to understand, and do justice to, all the factors in systematic inference, we must not neglect the ways in which these typical fathers of science made their contributions. Probably there is no better example for study than Galileo. "In 1581, while watching a lamp set swinging in the cathedral of Pisa, he observed that, whatever the range of its oscillations, they were invariably executed in equal times. The experimental verification of this fact led him to the important discovery of the isochronism of the pendulum." Here we note the *alert attention* ready to notice and meditate upon features of the surrounding world which would escape the vast majority, and the *experimental facility* which enables the analyst to separate out special factors and control their occurrence. During two years which he spent as a lecturer on mathematics at the University of Pisa, "he carried on that remarkable series of experiments by which he established the first principles of dynamics and earned the undying hostility of bigoted Aristotelians. From the leaning tower of Pisa he afforded to all the professors and students ocular demonstration of the falsehood of the Peripatetic doctrine that heavy bodies fall with velocities proportional to their weights." [1] Galileo was a keen observer, a suggestive interpreter, and a clever

[1] *Encyclopædia Britannica*, art. "Galileo."

inventor. He saw problems and had the ingenuity to meet them.

The Need of Analysis. The progress of science evidently depends upon all the processes by means of which facts are gathered, analyzed, and interpreted. Such collection of data involves selection, and this selection is facilitated by processes of comparison and analysis. *The purpose is to eliminate the irrelevant and accidental and to center attention on the important and essential.* Whatever aids in this task is of primary importance for systematic inference. Only after surface appearances are passed and irrelevant but confusing circumstances eliminated can the problem be defined and its probable answer suggested. Analysis by comparison and experimentation is the pre-condition of explanatory conceptions.

But we must never forget that the determination of what is important and relevant is no easy matter and is, indeed, the outgrowth of that prior knowledge for which the scientist has served an apprenticeship. Every problem requires a more or less special method and technique, and these reflect the ideas which are in the mind of the investigator. If he have the wrong notion, he may lose much time and labor to no account until some fact, or facts, makes him realize that he has been guided by a false conception. Logic can give no royal road to discovery. It can only abet the teacher of science in making the student conscious of the best methods, mental habits and ideals and warning him against the dangers of haste and dogmatism.

The Value of Technique and Instruments. The scientific man is on his guard against hasty conclusions.

He knows how easy it is to be led into errors of fact or theory by his interests and prejudices. It is this awareness of the difficulties confronting valid induction that leads him to collect so many facts and to use instruments wherever possible. "The technique of scientific inquiry thus consists in the various processes that tend to exclude over-hasty 'reading in' of meanings; devices that aim to give a purely 'objective' unbiased rendering of the data to be interpreted. Flushed cheeks usually mean heightened temperature; paleness means lowered temperature. The clinical thermometer records automatically the actual temperature and hence checks up the habitual associations that might lead to error in a given case. All the instrumentalities of observation — the various meters and -graphs and -scopes — fill a part of their scientific rôle in helping to eliminate meanings supplied because of habit, prejudice, the strong momentary preoccupation of excitement and anticipation, and by the vogue of existing theories. Photographs, phonographs, kymographs, actinographs, seismographs, phethysmographs, and the like, moreover, give records that are permanent, so that they can be employed by different persons and by the same person in different states of mind, i.e., under the influence of varying expectations and dominant beliefs."[1]

The attention of the scientist is more sustained and his observations are more systematic and unprejudiced than those of the untrained man. He possesses a mental technique of habits as well as the technique of instruments. But with modern methods of education, no one should be without some tincture of these methods and

[1] Dewey, *How We Think*, pp. 87–88.

attitudes. Logic seeks to inculcate those mental qualities which make for good thinking, such as caution, carefulness, thoroughness, orderliness, exactness, criticalness. In this sense, education is primarily the formation of logical dispositions, of "careful, alert, and thorough habits of thinking."

The Importance of Measurement. Definite knowledge has developed step by step with the power to measure phenomena. In the physical sciences at least, laws are equations between the measurable elements of phenomena. Hence the invention of new and more exact instruments of measurement has usually led to marked advances in the analysis of nature. "It would be a matter of great interest to trace out the dependence of this vast progress upon the introduction of new instruments. The astrolabe of Ptolemy, the telescope of Galileo, the pendulum of Galileo and Huygens, the micrometer of Horrocks, and the telescopic sights and micrometer of Gascoyne and Picard, Roemer's transit instrument, Newton's and Hadley's quadrant, Dollond's achromatic lenses, Harrison's chronometer and Ramsden's dividing engine—such were some of the principal additions to astronomical apparatus. The result is, that we now take note of quantities, 300,000 or 400,000 times as small as in the time of the Chaldeans."[1]

Experimentation. The devising of experiments is one of the important elements in a scientific investigation. An experiment works towards an analysis of nature and a control of the field under study which will make the important facts stand out clearly. The modern student desires to control the order of occurrence of his

[1] Jevons, *The Principles of Science*, vol. I, p. 315.

phenomena and to vary the factors which are present. Under such conditions, he is able to eliminate features and notice the effect, and to introduce new features and see what happens. Within certain limits, he can do with nature as he wishes and make it answer his questions. Thus experimentation is a method of controlled analysis, synthesis, and variation which enables man to make combinations that could never be observed free in nature.

There are essentially three advantages in experiment. We can vary the combinations and circumstances of things at will; we can produce factors at will which we might have to wait years for otherwise or even never obtain; lastly we can overcome the rigidity of things as they ordinarily present themselves. The following account of experimentation brings out very well its place in investigation: "All inductive methods rest upon the regulation of the conditions of observation and memory; experiment is simply the most adequate regulation possible of these conditions. We try to make the observation such that every factor entering into it, together with the mode and the amount of its operation, may be open to recognition. Such making of observations constitutes experiment."[1] Ingenuity in experimentation is certainly one of the prime qualifications of an investigator in the physical sciences. Such men as Faraday, Liebig, Pasteur, Darwin, and Davy were noted for their fertility in ideas and methods.

The first stage of a science is dominantly observational in its mode of gathering and testing facts. Many sciences, like geology, find it difficult to pass beyond this stage

[1] Dewey, *How We Think*, p. 91.

because of the nature of the material with which they have to deal. But the *ideal*, at least, in all the sciences is the testing of hypotheses by carefully planned observations. We are all so familiar with the use of experiment in physics and chemistry that we scarcely give it enough thought, but the application of experimental methods to biology and psychology is newer and not so familiar. Yet, in these fields, also, nature is being compelled to react to conditions which would be unlikely to occur apart from human purpose. It is evident, then, that experimentation is a method of investigation and that it is the servant of ideas which are seeking to find themselves verified.

The Use of Experiment in Biology. Experimental biology dates back only a few decades and is only now being developed to the extent it deserves. It has been found that growing organisms can be interfered with at various stages in a grossly mechanical way without preventing the production of normal forms. "A particularly striking case is that of Clavellina, an ascidian, that is to say, an animal organism of considerable complexity. 'You first isolate the branchial apparatus from the other part of the body (which other part contains heart, stomach, and most of the intestine), and then you cut in two in whatever direction you please. Provided they survive and do not die, as indeed many of them do, the pieces obtained by this operation will each lose its organization (becoming a mere sphere of cells devoid of specialized structure) . . . and then will each acquire another one, and this new organization is also that of a complete little Clavellina.' "[1]

[1] McDougall, *Body and Mind*, p. 240.

Experimentation in Psychology. James's picturesque description of the rise of experiment in psychology is well worth quoting in this connection: "Within a few years what one may call a microscopic psychology has arisen in Germany, carried on by experimental methods, asking of course every moment for introspective data, but eliminating their uncertainty by operating on a large scale and taking *statistical* means. This method taxes patience to the utmost, and could hardly have arisen in a country whose natives could be bored. Such Germans as Weber, Fechner, Vierordt, and Wundt obviously cannot; and their success has brought into the field an array of younger experimental psychologists, bent on studying the *elements* of the mental life, *dissecting* them out from the gross results in which they are embedded, and as far as possible reducing them to quantitative scales. The simple and open method of attack having done what it can, the method of patience, starving out, and harassing to death is tried. . . . There is little of the grand style about these new prism, pendulum, and chronograph-philosophers. They mean business, not chivalry."[1] The analytic character of experimentation comes out clearly in this quotation. The experimenter tries to dissect and abstract, and he is helped in his effort by instruments and methods of control.

The Statistical Method. In complex fields science has gradually been forced to methods of tabulation in which results are set down in their appropriate places and averaged. *The observation of specific things has given way to the gathering and arrangement of data.*

[1] *Principles of Psychology*, vol. I, p. 192.

Only after these data are collected and tabulated is there any attempt made to interpret them and to draw the proper inferences. Combined with statistical methods is the use of averages of various sorts, such as the arithmetical, the 'weighted,' the geometrical, the mode, and the median.

Statistical methods are particularly employed in fields where experimental analysis cannot be obtained. Thus there is much use of statistics in the social sciences and in such a field as meteorology. After the 'primary statistical quantities' are gathered, they must be arranged in tables in such a manner as to bring out important correlations. As we shall see when we take up the method in more detail, there are many possibilities of fallacy in both the gathering and the manipulation of statistics.

The Method of Graphs. When relations between variables are complex and cannot be easily intuited, resort is increasingly had to the use of graphs. In a graph, a curve is plotted to bring out in a spatial way the relation involved. In this way, the correlation becomes visible to the eye and is grasped without difficulty. The interesting thing is, that such projection of variable quantities often leads to the discovery of relations which the investigator would not otherwise have noticed.

Graphs are used in statistical investigation as a means of bringing out the import of the data in a vivid way. We are all familiar with this use of diagrams to illustrate the comparative populations of the various countries, or the relative sizes of their armies and navies, or the wheat production in different years. Investigations

in criminology are usually condensed by the aid of graphs. In brief, the graphical method is one both of investigation and of exposition.

The Function of Hypotheses. In our treatment of scientific method, we have thus far called attention to the more external processes and technique which science has developed. But we must never forget that these processes require an informing spirit to guide them. There must always be a purpose at work and some fairly specific problem in sight before this machinery begins to move. To become familiar with the technique and instrumentation of science does not make a creative scientist; nor is it enough to stare passively at test-tubes and microscopes. Facts are not objects to be collected like pebbles. They are not things which stand out in the environment with an external unity and identity of their own, but are dependent for their existence, in one sense, and certainly for their perception, upon the mind of the investigator. Facts do not pass into the mind ready-made, nor do they offer themselves. The scientist must be on the watch for them and must aid them to express themselves. His mind must be sensitive to them. Without mental activity, it would be impossible to gather facts. Why is this? It is founded on the very nature of voluntary attention. *We attend to what we are interested in; other things pass unnoticed.*

The facts connected with any investigation must be *relevant* facts. And this word gives us our clue. To be relevant is to be related to a purpose and a set of ideas. We must have some standard of selection, some way of telling what is important and what is unimportant.

There is a hazard in this, but the investigator must always take his chance. The warning that science gives by its history is general, and the individual scientist must make his own judgments and collect his own instances.

While the mind must be active in observation, it must be still more active, and certainly more creative, in experimentation. Some theory or hypothesis is always at the basis of an experiment. The experimenter has some idea of what he expects to find. The experiment is, in fact, usually devised consciously to test a conception which has gained favor. If oxygen is the essential factor in combustion, then its removal should prevent it. The resultant experiment is the crucial instance for the theory. If we are forced to deny a consequent, we are forced to deny the antecedent likewise, and to reject the idea which was gaining ground. It was in such a way as this that the phlogiston theory, which held sway before oxygen was discovered, was disposed of.

In a later chapter we shall have more to say of the origin and function of hypotheses. What we wish to stress just now is the activity of the mind in every stage of investigation. To leave out the mind is like removing Hamlet from the play.

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CHAPTER XVII

OBSERVATION AND FACT

The Need of Observation. Since the principle which should control all investigation is fidelity to relevant fact, we must emphasize the part played by observation. Mere speculation uncontrolled by fact is almost certain to lose touch with reality. It may lead to the construction of beautiful systems but these systems for all their splendor and subtlety are sure to lack value as means of interpreting the world in which we actually live. True systems of knowledge must be given a factual foundation. Fact is both the stimulus to, and the test of, knowledge. But fact can be obtained only by observation, direct or indirect. The investigator must either gather his data himself or else trust to the testimony of others. The precept which the logician feels called upon to enforce is that the observational side of investigations should be thorough and unbiased. Only in this way can data approach that objectivity which is the ideal of logical thought.

The Difficulty of securing Data. The sources of knowledge, so far as facts are concerned, are *direct* and *indirect.* The only direct way is by means of personal observation, and this involves perception. Even memory must be regarded as an indirect source since memory is the more or less correct revival of what has once been experienced. Another motive for classifying memory among the indirect sources of knowledge is the

greater danger of mistake. Slight, if not gross, errors in memory are so frequent that careful thinkers feel compelled to take the greatest precautions and to institute some method of record which will serve as a substitute for the details which are so easily forgotten or changed.

Conditions of Accurate Observation. The psychologist informs us that we notice only the things which interest us. Many stimuli come to our senses which never pass into consciousness because they do not awaken even a passing attention. This control of observation by interest is strikingly exemplified by the different accounts of events given by eye-witnesses. The records of the law courts are full of such conflicting testimonies on the part of eye-witnesses, and the experimental psychologist has been making experiments to study the degree of accuracy to be expected. The following example may make clearer the difficulty in securing trustworthy data when the series of events to be observed is complex: "A few years ago a painful scene occurred in Berlin, in the University Seminary of Professor von Liszt, the famous criminologist. The Professor had spoken about a book. One of the older students suddenly shouts, 'I wanted to throw light on the matter from the standpoint of Christian morality!' Another student throws in, 'I cannot stand that!' The first starts up, exclaiming, 'You have insulted me!' The second clenches his fist and cries, 'If you say another word —' The first draws a revolver. The second rushes madly upon him. The Professor steps between them and, as he grasps the man's arm, the revolver goes off. General uproar. In that moment

Professor Liszt secures order and asks a part of the students to write an exact account of all that has happened. The whole had been a comedy, carefully planned and rehearsed by the three actors for the purpose of studying the exactitude of observation and recollection. Those who did not write the report at once were, part of them, asked to write it the next day or a week later; and others had to depose their observations under cross-examination. The whole objective performance was cut up into fourteen little parts which referred partly to actions, partly to words. As mistakes there were counted the omissions, the wrong additions, and the alterations. The smallest number of mistakes gave twenty-six per cent of erroneous statements; the largest was eighty per cent. The reports with reference to the second half of the performance, which was more strongly emotional, gave an average of fifteen per cent more mistakes than those of the first half. Words were put into the mouths of men who had been silent spectators during the whole short episode; actions were attributed to the chief participants of which not the slightest trace existed; and essential parts of the tragi-comedy were completely eliminated from the memory of a number of witnesses."[1] In such cases there is hardly such a thing as pure perception. Suggestion, imagination, and inference shoot through what is called 'perception.' It is obvious that it is not very easy to find fact even when we wish to be faithful to it. But forewarned is at least forearmed.

In physical science the facts to observe are seldom as complex as in social matters. Yet the warning is not

[1] Münsterberg, *On the Witness Stand*, pp. 49–50.

out of place in the physical field. The chief advantage which the scientist has is that he can repeat his facts until he is satisfied that he has them correctly.

Haphazard observation occurs when there is no very definite purpose back of the series of perceptions. Things and events are then noted at random according to the interests and associations which happen to come uppermost. When such haphazard observations are examined, they are nearly always found to be too inaccurate or fragmentary to be of much value. *We see, then, that one of the prime conditions of good observation is definiteness of purpose.* The investigator must be on the outlook for facts of a known character or he will miss them. We are all aware that we are more apt to find a lost article if we know what to look for. The scientist who is carrying on an investigation has a fairly definite problem in mind which narrows the field of attention. This is one of the reasons why experimentation is so satisfactory.

Another condition of good observation is the possession of good mental habits. He who is on his guard against the influences constantly at work to distort observation is more apt to observe correctly. He will try to keep his mind open, to thrust prejudices and preconceptions aside, to allow facts repugnant to him, for one reason or another, as much weight as favorable facts; in short, he will try to keep his mind as receptive to all relevant facts as he is able. This neutrality of mind is harder to attain to than is generally supposed. It is an achievement of training and education.

Errors in Perception. Errors in perception are usually classified under two headings, 'mal-observation'

and 'non-observation.' Mal-observation is, again, of two kinds: addition to what is present and wrong arrangement of the parts. Non-observation is essentially omission. Let us consider these types of perceptual error in some detail.

We constantly add elements to interpret or enlarge what we actually perceive. This is not surprising when we remember that sensation is only a small part of perception. Association and suggestion are fundamental factors in perception, and we see what we wish to see or expect to see so long as the sense-factors and the general situation do not forbid us. When common sense says that seeing is believing, it is not aware of the part played by past experience and by desire in what we ordinarily take to be given. And this ignorance that perception is a product accounts for the dogmatism of common sense and for its perplexity when confronted by varying stories of the same events. In his book entitled *On the Witness Stand*, Professor Münsterberg illustrates the influence of mental factors in perception by the following instances: "In some Bowery wrangle, one witness was quite certain a rowdy had taken a beer-mug and kept it in his fist while he beat with it the skull of his comrade; while others saw that the two were separated by a long table, and that the assailant used the mug as a missile, throwing it a distance of six or eight feet. In another trial, one witness noticed at the seashore in moonlight a woman with a child, while another witness was not less sure that it was a man with a dog. And only recently passengers in a train which passed a courtyard were sure, and swore, that they had taken in at a glance the distinct picture of a

man whipping a child; one swore that he had a clean-shaven face, a hat, and was standing, while another swore that he had a full beard, no hat, and was sitting on a bench. The other day two most reliable expert shorthand writers felt sure that they heard the utterances which they wrote down, and yet the records differed widely in important points."

Non-observation is the overlooking of facts which ought to be observed. "It is exceeding rare to find persons who can with perfect fairness estimate and register facts for and against their own peculiar views and theories. Among uncultivated observers the tendency to remark favorable and forget unfavorable events is so great that no reliance can be placed upon their supposed observations. Thence arises the enduring fallacy that the changes of the weather coincide in some way or other with the changes of the moon, although exact and impartial registers give no countenance to the fact. The whole race of prophets and quacks live upon the overwhelming effect of one success, compared with hundreds of failures which are unmentioned and forgotten. As Bacon says, 'Men mark when they hit, and never mark when they miss.' We should do well to bear in mind the ancient story, quoted by Bacon, of one who in pagan times was shown a temple with a picture of all the persons who had been saved from shipwreck, after paying their vows. When asked whether he did not now acknowledge the power of the gods, 'Aye,' he answered, 'but where are they painted that were drowned after their vows?'"[1]

Causes of Erroneous Perception. A helpful

[1] Jevons, *The Principles of Science*, vol. II, p. 5.

classification of the causes of erroneous perception is into physical, physiological, and psychological.

Physical conditions are very important for correct observation. When the atmosphere is unclear, the scientist does not 'make observations,' for he knows that his data will be faulty. But since we are as much concerned with everyday reasoning as with science, we must note some of the difficulties due to external conditions. Objects change their colors according to the source of illumination. In a fog, objects look immense because our ordinary standards are removed. The dweller in the plain is unable for a long time to estimate distances in a mountainous region correctly. On the more technical side, we may mention the fact that instruments affect perception in various ways. Unless these sources of change are recognized and met, the data obtained may be misleading.

Physiological conditions are likewise important. Taste and color are affected by the condition of the body. Again, the registration of flashes of light in their time-relations cannot be made by perception because of a certain inertia in the nervous system. A glare of light is necessarily seen some fraction of a second after the eye has been stimulated. In delicate scientific work, the part played by the organism in perception is taken account of and automatic registrations are employed wherever possible.

Psychological factors are of more importance, but, since we have made some mention of them already, we will not dwell upon the topic. Misplaced attention leads to faulty observation. This cause is important in the activity of the prestidigitator, the false medium, the

pickpocket; they direct the attention of their subjects away from what is actually of most importance to them. Another instance of the significance of the psychological element is the value of training in the use of instruments such as the microscope. Again, the mind is affected by bad health. There is not the same possibility of controlled and tireless attention under abnormal conditions, however induced. An individual must be physically and mentally fit to be in a good condition for observation.

Summary for Perception. Seeing is not the simple and direct operation so often assumed. It has its conditions, physical, physiological, and psychological. Of these, the psychological are of the greatest importance for everyday life. Association and past experience in general play into perception, and its direction and content are determined by interest, desire, and other obscurer factors. Consequently, perception, even, cannot be taken at its face value when much is at stake; it must be tested and, if possible, repeated. We see much less than we suppose, and that is why observation is fullest when we have a definite purpose.

Observation in Everyday Life and in Science. Because the scientist has become aware of these dangers, he has made observation a more careful and intelligent operation than it is in everyday life. *The scientist makes an observation.* He knows what he is about and guards against dangers so far as possible. One advantage he usually has is that his observations can be repeated. An individual who is called before a court as a witness is only too painfully conscious that he did not know just what to observe; his attention was too

diffused and unguided to give definite results. He has, moreover, no records and must trust to memory with its attendant errors.

Memory and Facts. Memory is selective much as perception is. We are likely to remember only those events which we have expressly attended to; and a further elimination comes from the fact that we are much more apt to remember favorable than unfavorable things. This tendency to forget what is unpleasant was recognized by Darwin. "I had also," he writes, "during many years, followed *a golden rule* — namely, whenever a published fact, a new observation or thought came across me, which was *opposed* to my general results, to make a memorandum of it without fail and at once; *for I had found by experience that such facts and thoughts were far more apt to escape from the memory than favorable ones.* Owing to this habit, very few objections were raised against my views which I had not at least noticed and attempted to answer." [1]

Habits of mind or customary points of view furnish the centers for this unconscious selection. Events which cannot be made to fit into the scheme of values and ideas which is gradually established in the personality drop into the background and finally cease to appear in consciousness. Not only is there omission, there is also distortion and coloring of what is remembered. A person who writes his autobiography is apt to distort the facts of his life. That is why such narratives must always be checked where possible by external evidence. We must not say that Wagner and

[1] *Autobiography*, p. 87.

Disraeli lied, they simply deceived themselves. Who, moreover, can be perfectly frank, even to himself, about himself? Inhibitions of various sorts and our dramatic sense are at work upon the materials of our past life to make them more presentable.

Summary of Causes of Mistaken Memory. The following summary of the causes of mistaken memory may be helpful: —

We tend to remember what has usually happened and to forget the exceptional unless it is very striking.

We tend to remember events which are pleasant and fit in with the general trend of our lives and the drift of our ideas.

We tend to remember the heights and depths and to allow the more colorless events to lapse.

We tend to dramatize and develop events by an unconscious process of construction and inference.

The logician is aware that memory is not an intuition but a product, and that it must be tested by its inner consistency and by means of external evidence. The shorter the time which has elapsed, the more truthful it is found to be.

Testimony. Testimony is a still more indirect source of knowledge than memory. When we ourselves have not been the witnesses of an occurrence, we are compelled to have recourse to the statements of eye-witnesses. But a double danger of error is now present. We must be certain that those who claim to be were actually witnesses at first hand. Even so, they were subject to all the causes of mal-observation which we have already stressed, and, added to this, is the liability to change which their testimony is subject to in its passage

to our minds. Mal-observation, fallacious memory, inaccurate expression, misinterpretation, this is the series of distorting tendencies which makes 'fact' always more or less hypothetical. The consequence is that such tentative facts must be carefully examined before they are accepted. Methodical tests must be worked out and carefully applied; it is not too much to say that facts are critical judgments rather than something passively given.

Facts differ in Different Sciences. The data to which the investigator must adhere in his achievement of systematic knowledge vary from science to science. It is very important to bear this in mind, for, otherwise, it is impossible to understand differences in the methodology and technique of these various sciences. Inductive logic cannot be formal to the same extent that the logic of consistency is able to be. The geometrician secures his data by means of axioms and postulates which seem to him self-consistent, and which he relates to a spatial system lending itself to construction and manipulation. His methods reflect this foundation and are dominantly inspectional and deductive. The physical sciences obtain their data by active observation aided by experimentation. These observations lend themselves to repetition. The historical and social sciences have more difficulty in securing their data; in fact, data are more the end-terms of their investigations than the beginning. In their cases, we must distinguish between the crude material with which they start and the conclusions upon which they base their final interpretations. Between these stages lies a perfected technique of criticism and analysis. Thus the

character of investigation is modified by the nature of the problem, the aim of the investigator, and the kind of knowledge achieved. In spite of this variation, there is, however, the constant presence of the three elements to which we referred in the chapter introductory to this part of logic. There must be observation and collection of data, conjecture, systematic reasoning about the conjecture, and, finally, a return to observation. Only as the result of such systematic inference can knowledge be achieved.

What are Relevant Facts? We speak of the facts of a case and of relevant facts; how do we know what facts are relevant and what facts do actually belong to the case? They do not come labeled nor is the degree of their importance visibly stamped upon them. A prime condition of the acquisition of essential facts is guidance by some idea. We must know what we want, must have insight, judgment, discernment. Such discernment is, in part, the effect of familiarity with the field. The expert knows what to look for; he is able to narrow down the search to data of a certain character. There is in this sense of location what the psychologist calls an apperceptive or deductive element. Past experience is funded. We see and judge with the mind. Strictly speaking, there is no such thing as purely passive observation. There is usually a decidedly purposive factor at work. Guesses and conjectures arise to guide the direction in which to look for evidence. Especially is this the case when resort is had to experimentation. We may say, then, that all observation involves the selection of what is thought to be essential. The mind picks from the crowd of facts press-

ing upon attention those which are *judged* to be relevant. In other words, observation is in the service of a continuous process of interpretation and judgment.

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CHAPTER XVIII

THE ORIGIN AND USE OF HYPOTHESES

What is an Hypothesis? *An hypothesis is any conception by which the mind goes beyond the known facts and seeks to establish relations between data of testimony and perception, so long as that conception is one among alternative possibilities and is not considered a fact.* This definition, adapted from Bosanquet's larger *Logic*, brings out the essential position of an hypothesis as between accepted fact and accepted theory. It is a conjecture, a guess, a provisional explanation. Just because an hypothesis is conjectural, it is primarily a mental contribution whose aim is the solution of some problem. It is an interpretation or enlargement of what is given.

As a tentative solution of a problem, an hypothesis is an idea held before the mind in answer to a purpose, that of explanation. But this purpose cannot be fulfilled until many further steps are taken. The implications of the conjecture must be reasoned out and the attempt made to bring them into touch with a completer survey of the facts. Hence, a genuine hypothesis arises in fact and keeps in touch with fact throughout its history. Because of this continuous responsibility to, and control by, fact, the conjectural stage in investigation does not sin against the inductive principle of Fidelity to Fact. All that must be guarded against is that dreamy sort of speculation which soars into the

heavens and never seeks to return. "The truly scientific thinker has none of the spirit which says, 'If the facts do not agree with the theory, so much the worse for the facts.'"[1]

Kinds of Hypotheses. The nature of a particular hypothesis depends upon the character of the problem for which it is a tentative answer. Sometimes we are on the lookout for a fact, as when we are in search for a particular cause of an occurrence like a fire or a crime; sometimes we wish to establish a law of nature, such as a law of physical chemistry; sometimes we desire to conceive the structure of an empirical substance like rubber or protoplasm; sometimes we are pressing forward to some immense generalization like the principle of the conservation of energy or the theory of evolution. In each case, the mind must suggest ideas and then seek to establish them. The nature of the problem determines the character of the explanatory conjecture.

Are Hypotheses necessary for Science? At various times in its history, science has been skeptical of the necessity and even of the value of hypotheses. *During such periods, the ideal is to let facts speak for themselves.* It is supposed that, if facts enough are collected and tabulated, the principle which is sought will somehow stand out from them and force itself on the attention. "The natural goal of science based on this radically empirical method is to become a Science of Statistics so compiled and arranged as to force upon the methodical collector of observations the laws which the facts require to explain them. In this way, labori-

[1] Welton, *Manual of Logic*, vol. II, p. 86.

ous method takes the place of the scientific imagination and the happy idea."[1] This way of approach is often called the 'Baconian method' after Francis Bacon, one of the founders of modern logic. It represents a protest against an overspeculative tendency not fed on facts. It is, however, itself obviously another extreme. The relative importance of theory and fact varies from time to time and from subject to subject. It is now pretty generally held that the situation controls the amount of conjecture, but that a free play of the mind must always be present. It is psychologically impossible for principles to arise from facts without the collaboration of the mind. Neither observation nor conception is a passive event. Science is not a mere collection of facts.

A brilliant group of men who were at once scientists of recognized rank and thinkers championed the cause of hypotheses during the latter part of the nineteenth century, and their vindication of the controlled use of hypotheses has won general acceptance. These men worked in both the physical and the biological sciences. The following quotation from Huxley is typical: "It is a favorite popular delusion that the scientific inquirer is under a sort of moral obligation to abstain from going beyond that generalization of observed facts which is absurdly called 'Baconian' induction. But any one who is practically acquainted with scientific work is aware that those who refuse to go beyond fact rarely get as far as fact; and any one who has studied the history of science knows that almost every great step therein has been made by 'anticipation of nature,' that is, by the inven-

[1] Gibson, *The Problem of Logic*, p. 313.

tion of hypotheses which, though verifiable, often had little foundation to start with, and not infrequently, in spite of a long career of usefulness, turned out to be wholly erroneous in the long run."[1]

The Origin of Hypotheses. There are at least two conditions leading to the formation of hypotheses, the possession of facts bearing upon a problem and mental spontaneity. There is a constant interaction between these two factors the one creative and the other controlling and suggestive. As a rule, an hypothesis is not a sudden intuition, but a mental growth which matures in the fertile soil of accurate and extensive knowledge. When the history of any progressive investigation is written, analysis shows that there is a continuous transverse movement between data and conjecture. Conjecture guides observation and experimentation while the results of such further observation modify conjectures and select among them. We must always remember that the steps of induction which the logician separates out for study do not exist separately as mutually exclusive stages.

There is pretty general agreement that fruitful hypotheses are creations of constructive imagination. In its higher levels, such constructive imagination is a rare gift. A great scientist must combine many capacities; he must be a tireless observer and classifier; he must be able to organize his material and include it in one comprehensive survey; but he must also be able to seize that idea which will illuminate it and give it life and meaning. Tyndall was aware of the importance of this more personal factor of imaginative interpretation when he wrote: "With accurate experiment and observation

[1] *Methods and Results*, p. 62.

to work upon, Imagination becomes the architect of physical theory. Newton's passage from a falling apple to a falling moon was an act of the *prepared* imagination, without which the 'laws of Kepler' could never have been traced to their foundations. Out of the facts of chemistry the constructive imagination of Dalton formed the atomic theory. Davy was richly endowed with the imaginative faculty, while with Faraday its exercise was incessant, *preceding, accompanying, and guiding* all his experiments. His strength and fertility as a discoverer is to be referred in great part to the stimulus of his imagination. Scientific men fight shy of the word because of its ultra-scientific connotations; but the fact is, that, without the exercise of this power, our knowledge of nature would be a mere tabulation of co-existences and sequences."[1]

A Glance at the Psychology of Conjecture. The form of scientific imagination varies with the nature of the science. "No one will question that mathematicians have a way of thinking all their own; but even this is too general. The arithmetician, the algebraist, and more generally the analyst, in whom invention obtains in the most abstract form of discontinuous functions — symbols and their relations — cannot imagine like the geometrician."[2] The case seems to be that the nature of the materials is a factor of first importance; it is determining, and indicates to the mind the direction in which it is turned. Furthermore, some individuals are better able, by the very constitution of their minds, to work in a certain kind of material than in other kinds. Again,

[1] *Fragments of Science*, pp. 111–12.
[2] Ribot, *Essay on the Creative Imagination*, p. 237.

there are distinct differences in type when we come to consider the way in which the illuminating idea arrives. Roughly speaking, there are two types of creative minds, the intuitive and the reflective.

With the intuitive mind, the preparation is largely unconscious. The illuminating idea, when it does come, seems to burst upon the individual like a stroke of inspiration. There is a cry of 'Eureka' and the new view is born. Musicians and poets are more likely to have this type of creative imagination than are scientists; yet "Hauy drops a bit of crystallized calcium spar, and, looking at one of the broken prisms, cries out, 'All is found!' and immediately verifies his quick intuition in regard to the nature of crystallization."[1]

The reflective mind, on the other hand, realizes that there is a problem to solve and approaches its solution systematically. Such an individual gathers his material and broods over it. An interpretative idea gradually forms itself and is developed little by little. Perhaps it is soon rejected, and another grows up to supply its place. Kepler is a good example of the reflective mind. He devoted a large part of his life to trying new and strange hypotheses about the planetary movements until the day he discovered the elliptical orbit of Mars.

According to Ribot, the differences between these two types of minds are largely reducible to temperament and disposition. He works out the contrast as follows:—

Intuitive type	*Reflective type*
Ready-witted minds, excelling in conception, making the whole almost out of one piece.	Logically developing minds, excelling in elaboration.

[1] Ribot, *Essay on the Creative Imagination*, p. 247.

Intuitive type	*Reflective type*
Work primarily unconscious.	Patience the preponderating rôle.
	Work primarily conscious.
Actions quick.	Actions slow.

When we contrast these two types of mind, it becomes evident that logic has more to learn from, and more to convey to, the reflective type. But we must not forget that in any complex field of investigation to-day, the contrast is quite relative. There must always be patience and conscious effort.

Running all through logic is an almost moral element. The history of investigation proves that certain attitudes are favorable to invention and discovery while others are distinctly unfavorable. Creative imagination does not exist in a mind that is conventional and dominated by habit and routine. "Alertness, flexibility, curiosity, are the essentials; dogmatism, rigidity, prejudice, caprice, arising from routine, passion, and flippancy are fatal."[1]

The Value of Hypotheses. We have already said enough to indicate wherein the value of an hypothesis lies. It sharpens and guides the mental eye. The following anecdote from the life of Darwin brings out this feature very well: "Darwin tells of a geological trip through Wales which he took while a student at Cambridge, in company with Sedgwick, the professor of geology. It must be remembered that this was before Agassiz had come forward with his theory of a glacial period in the world's history. Darwin writes: 'We spent many hours in Cwm Idwal, examining all the

[1] Dewey, *How We Think*, pp. 105–06.

rocks with supreme care, as Sedgwick was anxious to find fossils in them; but neither of us saw a trace of the wonderful glacial phenomena all around us; we did not notice the plainly scored rocks, the perched boulders, the lateral and terminal moraines. Yet the phenomena are so conspicuous that, as I declared in a paper published many years afterward in the *Philosophical Magazine*, a house burnt down by fire did not tell its story more plainly than did this valley. If it had been filled by a glacier, the phenomena would have been less distinct than they are now.' "[1] As James put it, "The only things which we commonly see are those which we pre-perceive."

But an hypothesis is of value also because it begins a systematic analysis and synthesis of the facts. It is the beginning of an explanatory view which seeks to penetrate below appearances to the causes at work and their structure. When verified, hypotheses become theories and so enter as substantial elements in our knowledge of the world.

It is now generally admitted that an hypothesis which afterwards turns out to be false may have justified itself in a measure by its serviceableness. It is a nice question whether this relative serviceableness can be connected with an element of truth in its constitution or whether it is due to the guidance which it exercises. Any conjecture exerts a steadying control on attention and encourages the collection of facts, some of which may lead to its own dethronement. We must not forget, however, that an accepted hypothesis may prevent the consideration of truer ones and may in

[1] *Life and Letters of Charles Darwin*, quoted from Creighton.

this way retard progress. Certainly it is better to deal with true hypotheses than with erroneous ones. The Ptolemaic Hypothesis and the Newtonian Corpuscular Theory of Light are famous examples of long accepted hypotheses which were finally rejected.

The danger in the use of hypotheses is due to the tendency to dogmatism. All scientists are liable to "the partiality of intellectual parentage." One way to counteract this latent dogmatism which blinds the mental vision is to become conversant with the history of the various sciences. When we see theory succeed theory and even those which have stood the acid test of years of investigation modified in more or less essential points, we are less apt to retain our primitive assertiveness. Another way to prevent dogmatism is to keep before the mind at one and the same time a number of rival hypotheses. This is called the 'Method of Multiple Working Hypotheses.'

The Development of an Hypothesis. An hypothesis is barren which cannot be so developed as to imply verifiable consequences. The development of an hypothesis is sometimes called 'reasoning,' sometimes 'deduction.' What we do is to work out its consequences in the light of our knowledge of the system involved. Ideas which at first glance seemed plausible frequently turn out to be quite impossible. This mauling-over of ideas to get their bearings is essential. Let me first take a very simple case to illustrate the method. I see footprints on the beds in the garden which I have just planted. The idea comes to me that my young son has been disobeying my injunctions. But if these are his footprints, they will be only so large. I

look more closely and discover that they are larger. Thus my hypothesis has been developed and tested. This same process is apparent in all cases of reasoning. Detective stories are full of rather artificial examples of developed guesses which are valid or invalid as the case may be.

Science is replete with hypotheses which have lent themselves to development. Thus, Torricelli's theory that the air has weight implied that heavier liquids would not be raised so high in a vacuum as lighter liquids. This implication was tested and found to hold. Another implication was that the mercury would not stand so high on a mountain as on a plain. This also was verified. The implications of an idea can often be worked out only by one who is cognizant with the whole system of relations and facts within which the idea must fit. Thus, Foucault showed that it takes longer for light to travel in a dense medium than in a rare medium. But the Corpuscular Theory of Newton had the reverse of this for its implication, and was therefore disproved. The general form of reasoning about an idea is hypothetical: if this is so, then this other is so; but if this other is so, then this fact follows. The chain may be short or long, but it must eventually end in a datum theoretically open to observation.

The Proof of an Hypothesis. A distinction must be made between *verifying* an hypothesis and *proving* it. An hypothesis is verified so far as its logical consequences in the system of knowledge of which it claims to be part are harmonious with the facts. 'If *A* is *B*,' we may decide, 'then *C* is *D*.' We observe with or without experimentation and discover that *C* is *D*.

This agreement reinforces our inclination for the position that A is B. So long as there is no other claimant, we tend to regard the conjecture as true. *It cannot, however, be said to be proved in any final sense until it can be shown that no other hypothesis can account for the facts.* The logical situation involves the principle of the hypothetical syllogism. It will be remembered that it is impossible to affirm the consequent and draw a certainly valid conclusion. Because C is D, it does not follow that A is B. But it is impossible to eliminate all other possible theories because we can never be certain of working within a complete disjunction. What, then, determines our confidence in a successful hypothesis? Ultimately a *consilience of results.* When a very large field is covered and coördinated by a conjecture and no other plausible idea is in sight or seems likely to be advanced, we give our assent and regard the hypothesis as proved.

Fact, Theory, and Hypothesis. Facts are of two kinds, facts of observation and facts of belief. While it must not be forgotten that even facts of observation are really statements which are open to reasonable doubt, the judgments and tests involved are usually simpler than for the second kind of facts. This second class of facts are complex statements which have reached the stage of practical certainty. They are farther away from perception than the first and involve conceptual construction of an advanced degree. It is a fact of observation that a stick seems bent in the water, but a fact of assent that the earth revolves around the sun. Both classes have this in common that we give their members whole-hearted belief.

When an idea is first suggested and entertained by the mind, it is called an 'hypothesis.' When its consequences are deduced and verified, it is apt to be dignified by the name of 'theory.' When it has gained unquestioned acceptance, it is likely to be spoken of as a 'fact' or a 'principle.' Not so long ago, evolution was referred to as an hypothesis; then it became a theory; now it has attained the standing of a fact or a principle. We must remember that there is no court of adjudication, so that what is a principle to one may be an hypothesis to another.

Analogy as a Basis of Reasoning. The remark is often made that such-and-such a conclusion was reached by analogy. We admittedly *reason by analogy* from one field to another. Examining such reasoning, logicians point out that it consists of the passage from one thing or group of things to another because of a recognized resemblance between them. "Two things resemble each other in one or more respects; a certain proposition is true of one, therefore it is true of the other." [1] Now such a passage is usually tentative and is therefore of the nature of an hypothesis; this fact is our justification for treating the topic in this chapter.

The symbolic form of analogy is as follows: A resembles B in certain respects; A exhibits the character V, therefore B will exhibit the character V also. In other words, we tend to hold that if two things agree in certain respects they are apt to agree in others. All classification is really based on analogy in this broad sense; but in classification the similarities noted are many and fundamental and we are dealing with what

[1] Mill, *System of Logic*, bk. III, chap. XX, par. 2.

we have reason to believe are things of the same kind. There is, however, no decided break between what is clearly classification and what would be admitted by all to be mere analogy. "Where analogy is very close, and well tested, and familiar, as between cancer and cancer, or man and man, class-names have generally been invented. It is newly seen likeness, doubtful likeness, or likeness where the examples are rare, that we have to recognize as well as we can without the aid of class-names. And it is to these kinds of likeness especially that, as a rule, we give the name 'analogy.'"[1] It is not too much to say that all the reasoning in a new field bases itself upon analogies with more familiar domains. Thus the wave character of the propagation of light was inferred from the known fact that sound which also was reflected traveled in waves.

False Analogy. Analogies are very apt to be misleading. After all, things may be similar in this respect or that while differing in almost everything else. The points of similarity are often superficial and not correlated with the properties which are important for the problem in hand. Because a whale lives in the water and is shaped very much like a fish, it does not follow — as was at one time supposed — that it has the deeper structure of a fish. Because a municipality is a government, it does not follow that it must have the same type of organization as the Federal Government. An argument from analogy has a high degree of probability only when the points of resemblance are rooted deep in the nature of the two things and there is reason to believe that in the familiar field there is a connection

[1] Sidgwick, *The Process of Argument*, p. 40.

between the points of resemblance and the property which we wish to infer in the new field. A good instance of what must be regarded as a false analogy is Carlyle's argument against the representative form of government. "According to Carlyle, this kind of government is bound to fail, since, as he puts it, a ship could never be taken around Cape Horn if the captain were obliged to consult the crew every time before changing his course. A generalization is implied, something like, 'The sharing of power involves a lack of efficiency.' Granted that this holds true on ships, is it also true in government? The argument asserts that the two cases are alike, but it offers no proof that the difference in circumstances is immaterial. The apparent difference, however, is so great that caution is advisable. It may be that the lack of efficiency is due to the sharing of power under certain conditions peculiar to the management of ships."[1]

It is obvious that analogy must be controlled by analysis. False analogies always rest upon the disregard of differences. Thus analogy is a fruitful source of suggestions but these must be tested by a further survey of the facts and an estimation of their meaning. Resemblance may, as the saying goes, be only skin deep.

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[1] Bode, *An Outline of Logic*, p. 163.

CHAPTER XIX

THE DISCOVERY AND PROOF OF CAUSAL RELATIONS

How Experience comes to us. The individual's experience comes to him in an accidental, haphazard way, and it is only as the result of unconscious or semiconscious sorting and generalization that he reduces it to some show of order. Things are recognized and classified into kinds, and these kinds are supposed to have certain properties which can be predicted and counted on. Iron is malleable; coal burns; swans are white; apple trees yield apples after their kind, etc. By means of these classifications and generalizations, we are able to handle things and adapt ourselves to our environment.

But there is another feature of our experience which is equally important. We soon learn to note that events in which we are interested are controlled by conditions. If we want B to happen, we must see that A is added. If we want the garden to produce an abundant crop, we must add fertilizer in the proper proportions. If the manufacturer wishes to have steel of a certain degree of hardness, he must add manganese. *When any factor is known to be an invariable antecedent of some event or situation, and it is also known that this event does not occur in the absence of this factor, it is called a cause.* Such a cause is a means to an end, and is of practical significance. If we wish to achieve any result, we must see that all the conditions are present.

What are Causal Connections? It is usually said that science deals with the causes of things. What is meant by this expression? And what is a cause? A little reflection warns us that, because we are studying the actual methods of science, we have to do with a purely empirical problem. What the scientist is searching for is the condition of changes. Something happens, and he wants to know *why*. He is on the outlook for the factor which has led to the change in which he is interested. As soon as I add sulphuric acid to the test-tube which I hold in my hand, I begin to get a reaction, and I soon see a deposit at the bottom of the tube. Cause has, in short, to do with change in various parts of nature. We desire to know what disturbs the previous equilibrium and thus leads to novelty. A cause is, then, a factor which, so far as we can determine, is the occasion of the event which is selected as the effect.

When we come to reflect upon the relation between a cause and its effect, we note at once its temporal character. The entrance of the causal factor is the antecedent of the effect. But we are not willing to consider the entering factor a cause *unless its absence involves the absence of the effect*. It must be an indispensable antecedent. And it is not always easy to decide what factor is indispensable. We must know enough about the field to make a good guess or guesses, and we must then experiment to find out whether a fuller knowledge agrees with the guess.

Post Hoc ergo Propter Hoc. Even the older logicians were perfectly aware of the danger of false generalization. Because one event precedes another, we

must not jump to the conclusion that they are causally related. If we take the antecedent of an event in a purely temporal way, it includes the whole range of occurrences which can be dated before the event which we are interested in. A causal relation involves more than this temporal antecedence; it implies conviction that certain specific events located in some definite part of space are most intimately connected with the effect. He who is on the outlook for causal relations is seeking special strands of connection. He wants to find constantly repeated, invariable, dependable connections. His ideal is to analyze nature into uniformities which can be made the basis of prediction and control.

Early man had much the same tendency to treat temporal relations as causal that we have to-day, but he was far more credulous and hasty in his generalizations. What we call superstition and magic consisted largely of uncritical beliefs of this character. Chance association by contiguity or likeness was the foundation of accepted laws. The mind was ruled by ideas and made its selections accordingly with practically no thought of verification. It seemed very likely that the eating of a lion's heart would supply courage to the warrior; and had not *B* been unlucky when a rabbit crossed his path? The selection among events was personal and controlled by chance ideas and associations. To the critic the reply would have been, Why not? And there was a dearth of critics. It took time and hard experience to produce them.

The tendency to regard events as causally related just because one has been observed to follow the other has been christened '*Post Hoc ergo Propter Hoc*,'

i.e., 'After, therefore, Because of.' "The history of the progress of human knowledge shows, with increasing clearness, that better solutions are given in proportion as we recognize our own liability to be misled by first appearances. It is the testing and verification of theories, not the easy belief that they need no testing, that helps forward our knowledge of the ways of Nature." [1]

Mill's Methods. The human mind has gradually hit upon methods for discovering and testing causal relations. These were first sensed in the concrete by scientists in the pursuit of particular problems and were formulated some time after when reflection upon the general methods employed had arisen. The methods for determining causal relations were partially formulated by Herschel in his *Discourse on the Study of Natural Philosophy*, and were given their nearly final form by John Stuart Mill. For this reason, they are usually spoken of as 'Mill's Methods.' How can causal generalizations be discovered and verified? This is the problem around which they turn.

Mill formulated five methods, but the principles involved are essentially two. "The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon (any event), those with which it is really connected by an invariable law, are two in number. One is by comparing together different instances in which the phenomenon occurs. The other is by comparing instances in which the phenomenon does occur, with instances in other respects similar in which it does not. These two

[1] Sidgwick, *The Process of Argument*, pp. 132–33.

methods may be respectively denominated the 'Method of Agreement' and 'Method of Difference.'" These two principles would seem to follow from the very nature of the causal relation. If a cause is the indispensable antecedent of a phenomenon, can we not conclude (1) that the antecedent in the absence of which the phenomenon occurs is not the cause, and (2) that the antecedent in whose presence the phenomenon fails to occur is not the cause? It would seem possible, therefore, to eliminate factors which are absent when a particular effect occurs and to test those which remain by seeing whether their absence involves the absence of the effect. In this way, resort can be made to experimentation.

The Method of Agreement. "*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.*" Such is Mill's statement of the Canon of Agreement. It is possible to represent this canon schematically. Let X be the effect whose cause we wish to find, and let the accompanying circumstances be represented by *abcde*, *afklm*, and *aghno* in three cases of the appearance of the effect. Putting down these groups in the above order, we have:—

$$\begin{gathered} abcdeX \\ afklmX \\ aghnoX. \end{gathered}$$

Comparing these groups, we see that a is the sole factor which always accompanies X, and hence we feel the right to conclude that a is the cause of X.

Such a schematic arrangement, while it illustrates the

canon, is apt to be misleading because of its simplicity and artificiality. Much of the work of investigation must be done before any such neat little scheme can be constructed. But, after all, we are seeking just now to get the principle clearly in mind and we can note the setting and difficulties of the method afterwards.

Examples of Induction by Agreement. One of the classic examples in science of the application of this method is the discovery by Sir David Brewster that the material of mother-of-pearl has nothing to do with the brilliant display of color supposed to be characteristic of it. By accident, he took an impression of a piece of mother-of-pearl in beeswax and found, much to his surprise, that the colors were reproduced upon its surface. His interest being attracted, he carried on the experiment with other materials such as gum-arabic, balsam, etc. In each case, the colors appeared. The inference was obvious. The only feature these objects had in common was the form secured by impression from the mother-of-pearl.

When Sir Isaac Newton was investigating the property of physical objects called 'mass,' he had to determine that the material of which the body was made did not affect this property. In the same way, Galileo proved that bodies fall to the ground at a rate irrespective of their weight by the simple expedient of dropping bodies of different weights from the leaning tower of Pisa.

Let us suppose that an epidemic of typhoid fever has appeared in a city. In order to prevent its further spread, the health authorities will wish to locate the source of the disease. Past experience will suggest certain possible sources, such as the milk supply, the water

supply, the food, etc. Examining the milk supply, the authorities may find that the various patients secure their milk from different dairies. This fact makes the milk an improbable source. Turning to the food supply, they may find that no one article of food is obtained at the same store. The water supply, on the other hand, is common. This agreement will suggest to them very strongly that the infection was spread by contaminated water. If, furthermore, they find no other common articles of consumption, they will undoubtedly conclude that the water supply was at fault.

The Character of the Method. The Method of Agreement is more one of observation than of experimentation. As a rule, a large number of instances is necessary in order to make certain the inclusion of the cause among the factors noted. In practice, the method is preliminary and does not do much more than eliminate theoretically possible factors and narrow down the field for experimentation. It is an empirical way of approach to a problem and stresses observation. It does not get us far beneath the surface of things. "Thus varying the circumstances so as to bring out a common antecedent, though it does not end in exact proof, may indicate causal connection though it does not prove what the nature of the connection is. Roger Bacon's observations indicated that the production of rainbow colors was connected with the passage of light through a transparent globe or prism. It was reserved for Newton to prove by other methods that white light was composed of rays, and that those rays were differently refracted in passing through the transparent medium."[1]

[1] Minto, *Logic, Inductive and Deductive*, p. 325.

Difficulties confronting the Method. There are two important difficulties confronting this method. First comes the inability to determine in any dogmatic way what factors are present; and, added to this, the possibility that the effect may be produced by different factors on different occasions. Let us examine these difficulties somewhat closely.

It is so easy to ignore factors because they do not seem to have any connection with the effect. Hasty elimination of factors as the result of prejudice can easily lead to false results. Again, analysis is often too incomplete to enable the observer to recognize the constant factor. A very good instance of this oversight of the really important factor was the theory, held before the part played by micro-organisms in the propagation of disease was understood, that night air was the cause of malaria. It was noticed that people who caught malaria were generally exposed to the night air, but the fact that they were bitten by mosquitoes was not thought important and was therefore disregarded.

The fact that about the same effect can be produced by several causes is expressed by the phrase, 'plurality of causes.' Death, for instance, can be caused in many ways, and so can headaches and sickness generally. The common factor — when I take only a few cases — may not be the effective antecedent. My headache may be caused one night by smoking a cigar and, another night, by drinking too much coffee. We must conclude that the method of agreement is preliminary and that it is final in very simple cases only. The scientist regards it as a stepping-stone to experimentation; it leads to guesses which can be tested.

The Method of Difference. "*If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect or the cause, or an indispensable part of the cause, of the phenomenon.*" A more succinct statement of the canon, bringing out its experimental character, is as follows: "When the addition of an agent is followed by the appearance, or its subtraction by the disappearance, of a certain event, other circumstances remaining the same, that agent is the cause of the event."[1] It should be noted that only one factor should be changed. Great technical skill is often needed to fulfill this essential condition. The investigator must have control of his material.

Examples of the Method. Suppose we have been led to ask ourselves why it is that a feather does not fall to the ground as fast as does a stone. The idea may come to us that the presence of the air is the cause. If we can remove this and keep the other factors the same, we will be able to test our conjecture. The two instances will be as follows: —

	Antecedent conditions	*Effect*
1st instance:	stone and feather dropped in the air	stone falls first
2d instance:	stone and feather dropped in a vacuum	both fall together

This method is used primarily in physics and chemistry, for the material of these sciences lends itself to a literal analysis and synthesis under experimental con-

[1] Mellone, *An Introductory Text-Book of Logic*, p. 274.

ditions. It is less easy to apply it to organisms and to social conditions.

Warnings. When the method can be applied under experimental conditions so that only one factor varies, the only danger lies in the interpretation of the result. The factor may not be the only cause which is able to produce the effect. A plant may die when water is withheld, but it will also die when heat is removed. A positive effect often demands the coöperative presence of many conditions, and then the removal of any one of these will prevent the occurrence of the effect. Another difficulty lies in the possible presence of *counteracting causes.* A man who is very healthy may be able to resist infection, and so may a man who has had the disease and is now immune. So long as there is incomplete analysis of the situation, interpretation of the result of an experiment has its dangers. Only as further knowledge is gained, does the only possible interpretation begin to stand out.

When experimental conditions cannot be obtained, there is great danger that the two instances may differ in more than one point. "As an exemplification of this, we may cite an inference that was supposed to be warranted by the experience of one of our larger cities, which had voted to increase materially the price of liquor licenses. The police records, for the period immediately subsequent to the time when the new law went into effect, showed a distinct decrease in the amount of crime. By the method of difference we should naturally attribute this fact to the high license. But it was found that after the election the officials who issued the licenses had been much more careful

than before to issue licenses only to applicants of good moral character. This fact alone might account for the decrease in crime, leaving the higher license without effect upon the result." [1]

The Method of Concomitant Variations. "*Whatever phenomenon varies in any manner, whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*" The development of quantitative methods in modern science has made it possible to measure the variations of the important antecedents of an effect and to note corresponding variations in the effect itself. This mode of procedure is especially valuable when the conjectured cause cannot be removed completely but only decreased and increased. The method can be used either as a substitute for the Method of Difference or as a reinforcement of it when the interest of the experimenter travels to the study of the quantitative aspects of the causal relation.

Examples of the Method. The history of science is replete with instances of the application of the Method of Concomitant Variation. By the experiment of Count Rumford, heat was shown to be the effect of motion. "In this famous experiment, which disproved the material theory of heat, a blunt steel borer $3\frac{1}{2}$ inches wide was turned by horse power 32 times a minute inside a brass cylinder weighing 113 pounds. In two and a half hours the water surrounding the cylinder and weighing $18\frac{3}{4}$ pounds was heated from 60° F. to the boiling point. Only 4145 grains of

[1] Bode, *An Outline of Logic*, p. 139.

the metal were abraded. Rumford correctly concluded that this large amount of heat, which appeared to be inexhaustible, could not have been derived from the abraded metal, which at the same time had not lost any of its capacity for heat." [1] The First Law of Motion, Boyle's Law, the theory of the thermometer, the connection of air with the propagation of sound are all dependent, in whole or in part, upon the use of this method.

Warning. This method stresses the ratio between the change in the causal factor and the change in the effect. It has been found, however, that there is such a thing as *discontinuous variation.* Metals do not continue to expand at the same rate as the temperature increases, nor do they contract at the same rate as the temperature is lowered. Another point should be noted. It often happens that both of the quantities measured are co-effects of a third factor. Hence, where possible, the Method of Concomitant Variations should be supplemented by the Method of Difference.

The Joint Method of Agreement and Difference. "*If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*" The Joint Method is a combination of the Method of Agreement and the Method of Difference, and is especially applicable when experimental conditions are not

[1] Carhart, *University Physics*, pt. II, p. 4.

obtainable. It is a method of group comparisons. The investigator collects cases in which the desired effect is present and compares the instances to see in what they agree; he then collects other cases in which the effect is absent and compares these instances to see in what they agree. Finally, he examines the two groups to see wherein they differ. In this way, he approaches as nearly to the Method of Difference as can be done without experimentation. Without some control of the factors, the conclusion can be regarded only as tentative.

Examples. "The following instance illustrates the use of the Joint Method of Agreement and Difference: A large number of cases of typhoid fever occurred at about the same time in a college community. It happened that all those who developed the disease ate at a certain few fraternity and boarding-house tables. The water supply was first investigated. It was found that all these places used water from the same source. But it was also true that the other houses were supplied from the same source, so this possible cause was eliminated. The fresh vegetables were supplied from various sources; some of the places in which the disease was developed used one source, others a different one; moreover, the places in which the disease was not developed were supplied from the same variety of sources. The other food supplies came from various places and the Method of Agreement could not be applied so far as they were concerned, with one exception; it appeared that the milk supply was the same for all the places in which the fever was developed, whereas none of the places which escaped used milk from that source. The

inference was that the milk contained the cause of the disease. Further, it was found that when milk from this source was no longer used, no new cases of the disease appeared."[1] Another good example is to be found in the work of Darwin upon the cross-fertilization of flowers. In this investigation, Darwin placed a net about one hundred flower heads to protect them from bees. At the same time he exposed another hundred flowers to the bees. Here we have the two groups between which comparison is to be made. He obtained the following results: The protected flowers failed to yield a single seed, while the others produced 68 grains' weight of seed, which he estimated as numbering 2720 seeds.

These examples show that the Joint Method is really the complete application of the two principles of Agreement and Difference, and that any exhaustive investigation is bound to take this form. Another point stands out clearly. In any actual study the mind is always making conjectures and testing them by these methods. The scientist is not a merely passive tabulator. He knows that the cause must be present when the effect is, and that it must be absent when the effect is absent. Hence, he is on the constant lookout for factors which may fulfill these conditions.

The Method of Residues. "*Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.*" This method can be employed only when there is nearly complete information in regard to the

[1] Jones, *Logic, Inductive and Deductive*, pp. 98–99.

field under investigation. By means of this knowledge, we are able to eliminate causal couples until we are left with the few factors remaining for which causal relations have not been determined. In this way, past knowledge enables the investigator to narrow the field. After this is accomplished, he can observe more closely and often experiment. Like the Method of Agreement, the Method of Residues is usually a preliminary process which needs supplementation.

Examples. The Method of Residues comes out most clearly where quantitative methods are being employed. For astronomy, the classic example is the discovery of the planet Neptune. Certain perturbations in the movement of Uranus could not be accounted for by known gravitational forces. The natural inference was that some unknown planet was at work producing this added increment, the residual effect. Adams in England and Leverrier in France made this hypothesis independently of each other and calculated the probable place of the new planet. Following the directions of the latter, Professor Galle, of Berlin, found the new planet in almost exactly the place indicated. Another instance of discovery due to residual effects is that of the rarer elements in the atmosphere such as argon and helium. "In chemical analysis this method is constantly employed to determine the proportional weight of substances which combine together. Thus the composition of water is ascertained by taking a known weight of oxide of copper, passing hydrogen over it in a heated tube, and condensing the water produced in a tube containing sulphuric acid. If we subtract the original weight of the condensing tube from its final

weight we learn how much water is produced; the quantity of oxygen in it is found by subtracting the final weight of the oxide of copper from its original weight. If we then subtract the weight of the oxygen from that of the water we learn the weight of the hydrogen, which we have combined with the oxygen."[1]

This method is primarily one of quantitative analysis. It concerns itself with residual phenomena, with exceptions. Speaking of his father, Francis Darwin says: "There was one quality of mind which seemed to be of special and extreme advantage in leading him to make discoveries. It was the power of never letting exceptions pass unnoticed." Such a habit of mind is assuredly worth cultivation.

Remarks on Mill's Methods. There is pretty general agreement now among logicians that Mill's Methods can be employed only within a larger inductive setting. They are not rules which can be applied mechanically to new and complex fields. The mind must be active in conjecture, interpretation and analysis. In the first part of his *Logic*, Mill is far more Baconian than he is in the latter part. The following quotation gives all the correction that is needed and reveals how the Methods are taken up into the concrete process of investigation: "The process of tracing regularity in any complicated, and at first sight confused set of appearances, is necessarily tentative: we begin by making any supposition, even a false one, to see what consequences will follow from it; and by observing how these differ from the real phenomena, we learn what corrections to make in our assumption." Increasing

[1] Jevons, *Lessons in Logic*, p. 254.

analysis, working hypotheses, gradual elimination of irrelevant factors, and final decision as to the essential connections are the steps in induction.

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CHAPTER XX

STATISTICS

In many fields of investigation it is impossible to apply the traditional methods. The problem is of such a character and covers such a wide territory that individual observation cannot furnish the factual material. Furthermore, experimental methods cannot be applied. What is needed is a very broad survey of the facts and a clear tabulation of the results obtained. Out of this necessity has arisen statistical science. "It would be a man of exceptional mnemonic power who could, after listening to the reading of two lists of one hundred items each stating the names and wealth of the respective inhabitants of two villages, give any intelligent opinion as to the comparative riches of the two communities. If this is true for such small groups as this, it evidently would be utterly impossible to make comparisons of the wealth of great nations without some manner of reducing the mass of separate facts to a simple whole. The same would, of course, be true in the case of any other phenomena involving large numbers. What could one understand of the amount of lumber contained in a forest from a description of the separate trees? How could one compare the climates of different localities by a study of their daily weather records? It is for the purpose of simplifying these unwieldy masses of facts that statistical science is useful. It reduces them to numerical totals or averages which may be abstractly

handled like any other mere numbers. It draws pictures and diagrams to illustrate general tendencies and, thus, in many ways adapts these groups of ideas to the capacity of our intellects." [1]

If the student will recall the discussion of "Classification," he will remember that it was regarded as a process which enables us to handle the complex world of individual things in which we live. Could we not treat things as kinds and know what to expect from them, we should be unable to adapt ourselves to our environment by using our past experience. Now, statistics is a way of dealing with particulars which can be counted and which are so numerous that we need to achieve some bird's-eye view of them before we can begin to interpret them. It is a way of collecting and organizing data which are important in masses rather than individually. "The proper function of statistics," writes Bowley, "is to enlarge individual experience." Originally the workers in statistics concerned themselves almost entirely with facts respecting the condition of the people in a state, but now data in biology and astronomy are also investigated by statistical methods.

We have frequently pointed out that science always involves *analysis*. Mental work must be done upon the facts before they yield a meaning and can be applied to the solution of a problem. This element stands out very clearly in the following definition of statistics: "The science of statistics is the method of judging collective natural or social phenomena from the results obtained by the *analysis* of an enumeration or collection of estimates." [2]

[1] King, *Elements of Statistical Method*, p. 24. [2] *Ibid.*, p. 23.

The stages in a statistical investigation are roughly as follows: (1) The setting of the problem; (2) the collection of the material; (3) its tabulation; (4) the summary; (5) a critical examination of the results. Let us look at these stages somewhat closely.

"Special importance, it seems, must be attached to the question of what are the objects of any particular statistical record. *It is not enough to observe the facts at random.* Most of the records of statistics, although not all of them, are deliberately arranged to be kept by a community for the purpose of the information of the whole community. . . . The object, then, of any particular statistical record is the first thing about it to be studied, and both the Government in making the records and the students who come to use them afterwards should have the most distinct ideas upon this point."[1] This point is stressed by all statisticians. "The first thing upon which the statistical investigator, when beginning his work, must decide is the exact nature of the problem which he desires to solve. Even a slight change in its scope or form may require an entirely or partially different method of procedure. If, for illustration, a person wishes to begin a study of comparative wages in order to demonstrate some general theory or proposition, he must first decide as to whether the requirements of his problem demand a knowledge of money wages or real wages. Next, he must be sure as to whether he needs to know the wages paid for a definite amount of effort, for making a certain product, or for working a certain length of time, or whether the inquiry relates to the income of the working man him-

[1] Sir Robert Giffen, *Statistics*, pp. 4–5.

self per year or to the total income of the man and his family for the same period. Each of these problems is a distinct one and would require entirely different modes of determination."[1]

Definition of the problem also involves definition of the unit. What shall we mean by 'unemployment'? by 'person'? by 'a farm'? by 'sickness'? Just as classification and its principles are important for statistical investigation, so is definition. Thus a recent work on *Unemployment* gives a chapter to the "Meaning and Measurement of Unemployment" and devotes much of its space to the working-out of a definition. "Are we, for example, to include among the unemployed those who are idle because they do not want to work? Are we to include sick persons, or workmen out on strike, or the various classes of individual who are, for one reason or another, 'unemployable'?" Just as an idea must not be allowed to change its meaning in the course of an argument, so the unit must not be altered in the course of an investigation. The prerequisites of clear thinking are essentially the same for argument of a deductive character and for investigation.

Before the collection of the material is begun, the problem must be studied in all its details. "Problems, factors, units, questions, schedules, enumerators, tabulation, methods of work, time, expense, etc., are among the items that must be carefully gone over in minute detail." The methods of collection must be determined by experience with due regard to the characteristics of the particular problem. The field must be defined and must be as wide as possible. If questions are used, these

[1] King, *Elements of Statistical Method*, p. 39.

must be comparatively few in number, must be simple enough to be readily understood, and must not arouse prejudices. As in all the other divisions of science, personal judgment cannot be eliminated. Investigation is not a mechanical process.

"With the collection of statistical data, only the first step has been taken. The statistics in that condition are only raw material showing nothing. They are not an instrument of investigation any more than a kiln of bricks is a monument of architecture. They need to be arranged, classified, tabulated, and brought into connection with other statistics by the statistician. Then only do they become an instrument of investigation, just as a tool is nothing more than a mass of wood or metal, except in the hands of a skilled workman."[1] The figures must be grouped in tables under appropriate headings in such a way as to bring out the desired percentages and correlations. "The power to analyze a table, interpret the results correctly, and state the conclusions lucidly and succinctly is one of the characteristics indispensable in a good statistician."

The Law of Statistical Regularity. It has been discovered that *a moderately large number of items taken at random from a very large group, are almost sure, on the average, to have the characteristics of the larger group.* "Thus, if two persons, blindfolded, were to pick here and there three hundred walnuts each from a bin containing a million nuts, the average weight of the nuts picked out by each person would be almost identical even though the nuts varied considerably in size." Corresponding to this law is its corollary, the

[1] Mayo-Smith, *Statistics and Sociology*, p. 18.

law of inertia of large numbers. Logicians have been accustomed to speak of this law as the constancy of averages. "If we take a succession of periods, and divide the total number of any kind of event by the number of periods, we get what is called the average for that period: and it observed that such averages are maintained from period to period. Over a series of years there is a fixed proportion between good harvests and bad, between wet days and dry: every year nearly the same number of suicides takes place, the same number of crimes, of accidents to life and limb, even of suicides, crimes, or injuries by particular means: every year in a town nearly the same number of children stray from their parents and are restored by the police: every year nearly the same number of persons post letters without putting an address on them." [1]

Dangers in the Use of Statistics. Great care must be taken in the use of statistics. It has been said that "you can prove anything by statistics." And this skepticism reflects the constant misuse of figures. Every stage in the development of tables is liable to its peculiar errors which resemble those which face all investigation. Prejudice, inaccuracy, faulty observation, mistakes in copying, wrong interpretation, all exist as possible sources of falsehood. Another cause of error is the lack of recognition given to the difference between quantity and quality. "In sociological science the importance of differences of quality is enormous, and the effect of these differences on the conclusions to be drawn from figures is sometimes neglected, or insufficiently recognized, even by men of unquestionable

[1] Minto, *Logic, Inductive and Deductive*, p. 351.

ability and good faith. The majority of politicians, social 'reformers' and amateur handlers of statistics generally are in the habit of drawing the conclusions that seem good to them from such figures as they may obtain, merely by treating as homogeneous quantities which are heterogeneous, and as comparable quantities which are not comparable. Even to the conscientious and intelligent inquirer the difficulty of avoiding mistakes in using statistics prepared by other persons is very great. There are usually 'pitfalls' even in the simplest statistical statement, the position and nature of which are known only to the persons who have actually handled what may be called the 'raw material' of the statistics in question; and in regard to complex statistical statements the 'outsider' cannot be too careful to ascertain from those who compiled them as far as possible what are the points requiring elucidation."[1] So frequent a use is made of statistics now that the logician feels it a part of his duty to stress these warnings. The following example may serve to indicate how easily fallacy may creep in: "An example of such a fallacy, due to the use of erroneous factors, was furnished by a newspaper in a discussion of the American navy during the Spanish-American War. It was stated that the death-rate in the navy during the war period was only nine per thousand, while in the city of New York for the same period the death-rate was sixteen per thousand. The conclusion was drawn that it was safer to be a sailor in our navy in war-time than to live in New York City. A little reflection, however, will convince one that such a conclusion is not warranted

[1] *Encyclopædia Britannica*, art. "Statistics."

by the figures given. In obtaining this ratio, the total number of deaths was taken as the numerator in each case and the denominators were respectively the total number of persons living in New York City and the total number of sailors in the navy. But, as a matter of fact, *these numbers were wholly incomparable.* It is a well-known fact that the death-rate is very high among young children and among old people. But the personnel of the navy is composed almost wholly of young men in the prime of strength and vigor. Not only this, but each must pass a strict examination to show that he is healthy and robust. Thus, the weak and diseased are eliminated. Evidently, the facts would require that the death-rate in the navy be compared with the death-rate of a similar picked body of men in New York City before any legitimate conclusions could be drawn regarding the comparative chances of death in the two places." [1]

The Value of Statistics. There are *three* main ends subserved by statistics: Its employment contributes to a descriptive survey of a field which cannot be otherwise grasped. Figures are then merely a method of expressing facts, as the amount of wheat produced, the quantity of exports, the amount of the national wealth. Such information is often of the greatest significance. We may wish to compare one country with another, or the same country at different times. Very frequently the data obtained can be used to test a theory which has been worked out more or less deductively. Wherever there is induction, there must be appeal to relevant fact; and in many fields, statistics is the only

[1] King, *Elements of Statistics*, p. 40.

means of securing such facts. In the second place, the discovery of averages which are fairly constant enables man to predict the general run of cases in the future. As we shall see in the next chapter, the theory and practice of life insurance are founded upon the existence of such averages which are being continually developed and rendered more accurate. In the third place, tables of statistics often reveal correlations which would not otherwise have been discovered because of the complexity of the material and our inability to analyze it into distinct strands of causal uniformities. When data are properly arranged, the field is condensed and simplified and it is possible to note relations which would have remained hidden. The use of graphs is often an aid because they make a direct appeal to the eye. An example of a correlation which suggests the presence of causal relations is the fluctuation of the number of births in a country correspondent with the food prices of the previous year. Let us consider this third function of statistics a little more fully.

The principle underlying the deduction of causal relations from correlations is the assumption that if the events are unconnected, their frequencies should not fluctuate together nor coincide beyond a certain amount. Bain formulated the following rule: "Consider the positive frequency of the phenomena themselves, and how great frequency of coincidence must follow from that, supposing there is neither connection nor repugnance. If there be greater frequency, there is connection; if less, repugnance." Repugnance is usually spoken of in statistics as *negative correlation*. An example would be the relation between vaccination and small-

pox. In delicate cases, it is very difficult to estimate the degree of coincidence between non-causal events. How often, for instance, must dreams and their apparent fulfillment coincide to pass beyond mere coincidence?

Variation in averages is a fruitful source for the discovery of causal relations. If we assume that the constancy of the averages is due to the maintenance of the relative proportions of the same factors in the field under investigation, then any change in the average must be due to an alteration in this proportion or to the entrance of a new factor. A study of the geographical distribution of suicide is a good instance of the suggestive character of comparative statistics. We immediately ask ourselves why one country leads, what can be the peculiar conditions which cause more people to commit suicide there. It will be noted that such comparison involves a near approach to the method of differences. But experiment is impossible. When an average begins to change, the investigator seeks the presence of some new cause or the increase of some cause already operative. "The number of homicides in the United States in 1894 far exceeded the annual number observed for the years preceding. This discrepancy is easily accounted for by the fact that the natural number was swollen by the deaths caused by the strikers and rioters in the month of July of that year. So also a marked departure from the annual death-rate of such a city as New York is at once an urgent suggestion to the Board of Health to start investigations that will unearth the hidden cause that one is constrained to believe must be present. Such causes as defective drains, prevalence of epidemics, etc., are again and

again found to accompany an increase of the average death-rate." [1] The principle involved is essentially that of concomitant variation.

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[1] Hibben, *Logic, Deductive and Inductive*, p. 344.

CHAPTER XXI

PROBABILITY

Abstract Laws versus Concrete Events. While the physical sciences are chiefly interested in the discovery of universally valid causal laws and in the construction of illuminating general principles and theories, problems of another sort come to the front in practical life, the applied sciences, and those fields of research in which experimental control is impossible. Where experiment is possible, factors can be eliminated as irrelevant or confusing, and the remaining elements can then be analyzed into couples uniformly connected. But where such experimental control and *simplification* is out of the question, causes counteract one another in the most complex ways or else combine to produce effects which are not deducible from any one of them. In other words, such causal uniformities as we discover by experiment are blurred when they enter this whirligig of a world, and it is with the greatest uncertainty that we predict the course of events.

In science, the investigator usually passes from effect to cause; in everyday life, the problem is to pass from the play of events to the probable result. What it is now desired to ascertain is what will probably happen in the tangled skein of events which cannot be completely analyzed beforehand. Will the country be prosperous a year from now or plunged in a panic? Will this bank be solvent or will it fail? Will the

crops be good this year? Such questions make us realize the complexity of things and our inability to deduce events from the few uniformities we have as yet discovered. Most laws have their exceptions because the essential factors are masked by the interplay of their fellows.

Where Certainty is possible. Certainty as to the future is possible only when all the factors whose operations will produce it are known. Moisture causes plants to grow when it is combined *in the proper proportion* with heat and sunshine and good soil. It is evident that quantities are important if we are to predict. Thus, we must have knowledge of the essential conditions, qualitative and quantitative, of an event and have assurance of their presence before we have the logical right *to predict an event with certainty.* In a carefully conducted experiment in chemistry where quantities and qualities are both under control, expectation is based on exact knowledge, and all that we need to assume is the general principle of the Uniformity of Nature. Again, prediction is possible in astronomy where the forces at work are relatively few in number and measurable. In this realm of giant masses and gravitational forces acting according to well-known laws, it is possible to foretell the relative positions of bodies and so arrange in tables, centuries ahead or behind the present, those eclipses and juxtapositions which are of human interest. But when the antecedents cannot be analyzed in this fashion and are known to be more fluctuating in their permutations and combinations, prediction loses certainty and drops to probability.

The Meaning of Probability. Probability at-

taches to particular facts and to generalizations. We use the term when we feel that the balance of evidence is in favor of a principle or an individual fact even though it has not been raised above reasonable doubt. So far as principles are concerned, their probability reflects the degree to which they are supported by the collected evidence. It may be that all the evidence thus far attained is favorable but that we do not regard it as extensive enough; or it may be that the evidence is conflicting. When we are concerned with the probability of the occurrence of an event, on the other hand, we are attempting to make an estimate of our admittedly insufficient knowledge to determine whether it is in favor of, or against, this particular occurrence. We temper the degree of our expectation in accordance with such relevant knowledge as we can gather. In contrast with certainty, we may say that probability is the degree of expectation which is judged to be warranted by the facts. We say that an event is probable, or quite probable, or very probable, or extremely probable.

Probability and Chance. Probability must not be confused with objective chance. The principle of the Uniformity of Nature is essentially the denial of anything like real chance in the world. But nature is so complex that we are not able to note and analyze all the conditions of the majority of events. Causal factors reinforce and counteract one another in unforeseen ways, and the actual result can be learned only from experience. That I turned up an ace of spades the last time I cut for the deal was, so far as my knowledge went, a case of chance. In short, chance is a term ex-

pressive of my ignorance of what is taking place. "There is no doubt in lightning," writes Jevons, "as to the point it shall strike; in the greatest storm there is nothing capricious; not a grain of sand lies upon the beach but infinite knowledge would account for its lying there; and the course of every falling leaf is guided by the same principles of mechanics as rule the motions of the heavenly bodies." [1]

Three Kinds of Estimations. Roughly speaking, we may distinguish three kinds of probability-estimations. The first is empirical and largely qualitative; the second deals with averages and is closely connected with the development of statistics; the third is of a deductive, mathematical type. What logic is interested in is the nature of the induction and the dangers run.

Empirical, or Non-Quantitative, Probability. As regards most events, we do not keep any exact record of how things have happened in the past, but trust to a general impression, to a sort of cumulative estimation expressive of past experience. This judgment is then related to the particular circumstances of the case in hand and a feeling of probability arises in consciousness. Suppose that you hear that a person whose name is not given has met with an accident in the city of Washington. The idea may come to you that it may be your brother who lives there; but the idea is sure to be dismissed unless other information points in the same direction. Suppose that the accident happens in a government building where he works, and that the description of the individual tallies somewhat with his

[1] *Principles of Science*, vol. I, p. 225.

appearance, your passing idea begins to be entertained more seriously and you await some letter or telegram. If it does not come within a day or so, you dismiss the suggestion. You know that there are many government employees and that rough descriptions apply to large numbers of individuals equally well.

Probabilities based on Averages. A frequently employed method of calculating probabilities is the study of averages for classes. Sometimes such a study is non-statistical, but it readily passes into the stage of statistics. An individual may calculate that he has many more years to live because he is healthy and fairly young and is working in a profession where the risks are small. "This type of reasoning is duplicated in many other instances. When we mail a letter, we count pretty confidently upon its safe arrival. From time to time, letters get lost, through carelessness, in railroad wrecks, or as a result of other causes, but the number of those is so small a proportion of the total number that we treat it as practically a negligible quantity. For the same reason we leave out of account the possibility that at some time we shall be struck by lightning, or, when we make a journey, that our train will be wrecked, or that our home will be destroyed by an earthquake, or that a stranger of whom we make inquiries as to directions will be insolent or show annoyance."[1]

The best familiar example of the calculation of probabilities for a class is to be found in the field of life insurance. Tables are worked out for the various trades and professions and for individuals of different ages.

[1] Bode, *An Outline of Logic*, p. 150.

Any applicant is classified and his probable length of life estimated by reference to the group in which he is classified. A danger of fallacy lurks in this method when its meaning is misunderstood. It must be borne in mind that such probability only applies to the class and not directly to the individual. We are not able to predict which individuals will die, but only that about so many out of every thousand of a certain age will die. It is upon the constancy of these averages that the company calculates the necessary premium.

Estimates of probability are made more accurate by deepening the classifications made. The greater the number of essential characteristics that are included, the smaller and more homogeneous the class. Thus, teachers and ministers are known to have a far longer average of life than miners and textile workers. Statistics have shown that the average length of life of the rich is greater than that of the poor. It has been estimated that in England the average duration of life among the wealthy classes is from fifty-five to fifty-six years, while in the working classes it falls to twenty-eight years, or even lower.

A word of warning may, perhaps, be advisable at this point. Probability of this objective sort is based on the use of statistics. Now the growth of statistics has shown a constancy in the number of crimes committed in a certain country in the course of a year, in the number of suicides, in the number of fires, etc. In the preceding chapter, we spoke of this regularity as the 'law of inertia of large numbers.' The meaning of this law has been much misunderstood, many fearing that it implied a kind of mechanical necessity at work

in society corresponding to the 'reign of law' in nature. It is well to bear in mind that such expressions are metaphorical and that we must penetrate behind them to the exact facts of the case. Laws are human formulations and do not reign in the physical world by divine right or otherwise. In the same way, statistical constancies are only formulations of data and do not imply any fate lying back of, and controlling, society. The following passage from Quetelet, the famous Belgian statistician, is worth quoting in this connection: "Amongst the facts disclosed in my book, the one which has given rise to most alarm is the constancy of crime from year to year. By a comparison of numbers, I believed I had data for inferring, as a natural consequence, that in a given country, under the same conditions and influences, we might expect a repetition of the same facts, a reproduction of the same crimes and the same condemnations. But how was this received? A crowd of timid people raised the cry of fatalism! Now, what do the facts teach us? This, simply, — that in any given state, subject to the influence of the same causes, the effects will not differ appreciably; they will oscillate more or less about some mean. Now, mark what I have said: subject to the influence of the same causes; so that, therefore, if those causes change, the effects will be likewise modified. But, since the laws and principles of religion and morality are the source of the influences in question, I cherish not only the hope, but — what you perhaps do not — the deepest conviction even, that society can be reformed and ameliorated." The clear-sighted student will realize that we are here skirting some of the most interesting problems of philosophy.

What it is important and easy to realize is that constancy in averages does not involve fatalism.

The Mathematical Treatment of Probability. We can divide the mathematical treatment of probability into two parts, the conditions for the application of a calculus of probability and the rules for estimating probability.

The first condition for the application of a calculus of probability to events is the possession of knowledge sufficient to assure us that some one or other of a definite number of ways of happening are possible, but not sufficient to inform us in which particular way the event will occur. There must be this peculiar mixture of knowledge and ignorance. "For example, if a penny is tossed it will fall with either head or tail uppermost. Now, which will be uppermost in any particular throw will be exactly determined by such conditions as the position of the coin at starting, how it is grasped in the fingers, the force and direction of the twist, etc. But what special form these conditions will take we are totally ignorant."[1] The *second* condition is that the same general set of conditions and antecedents must be present and operative during the whole course of the events for which a calculation is being made. For instance, to substitute another coin with a curve on one of its surfaces for the original one would make a calculation impossible. Both of these conditions are secured in games of chance. Since it is not the function of the logician to enter into the mathematics of probability in detail but only into the logical theory at its foundation, we shall consider only the simplest cases.

[1] Welton, *Manual of Logic*, vol. II, p. 167.

The probability of a *single* event is expressed by a fraction whose numerator is the number of favorable alternatives, and denominator the total number of alternatives. Since a coin has only two sides, the probability of getting heads in any one trial is $\frac{1}{2}$. We say that the chances for heads and tails are equal and that the probability of each is $\frac{1}{2}$. In the case of dice, the number of sides is greater and, accordingly, the probability for any one side is less. The denominator is 6 while the numerator is 1 for any one side. This is expressed in mathematical form by saying that the probability for each side is $\frac{1}{6}$. The method of reasoning in the case of playing cards is essentially the same. The total number of possibilities is 52 and the chance for any one card is $\frac{1}{52}$. The chance of getting an ace would be $\frac{4}{52}$.

There are two rules with *compound* events according as such events are (1) independent or (2) dependent on one another.

The probability of obtaining any combination of independent events is the product of the probabilities of the several events. Let us take the throwing of dice as an example. "If one die is thrown the probability that it will fall with the side bearing six pips uppermost is $\frac{1}{6}$, as the die has six sides, and the probability of being uppermost is equal for them all. If a second die is thrown the probability that in that throw six will be uppermost is also $\frac{1}{6}$; consequently the probability that six will be thrown in each of two throws is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. In this case, it is obviously immaterial whether two dice are thrown simultaneously or whether the same die is thrown successively." [1] Obviously the number of pos-

[1] Welton, *Manual of Logic*, p. 174.

sibilities is 36 and only one of these is the combination desired.

If two events are mutually exclusive, the probability of getting one or the other is the sum of their independent probabilities. Thus, in throwing dice, the probability of getting a six or a five is $\frac{1}{6} + \frac{1}{6}$, or $\frac{1}{3}$. The principle is the same as for simple events.

We come now to compound events in which the occurrence of the first affects the probability of the occurrence of the others. *In such cases the probability of the compound event is the product of the probability of the first with the probability of the second as affected by the first.* "For example, what is the probability of drawing two white balls in succession, without replacing the first drawn, from an urn containing two white balls and one black one? If a black is drawn first, the estimate is rendered impossible. The probability of drawing a white first is $\frac{2}{3}$. If it is drawn, the constitution of the urn for the second draw is modified by the first draw. There are now only two balls, one white and one black, in the urn; and the probability of drawing the white is $\frac{1}{2}$." Therefore, the probability of drawing two whites is $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

Mistakes in interpreting Probabilities. Mistakes are often made in the interpretation of probabilities. Probabilities based on averages, as in life insurance, do not give us any information in regard to any particular individual. "Statistics, from the very nature of the subject, cannot and never will be able to take into account individual cases."[1] When we come to deductive, or mathematical, probability, other warnings

[1] King, *Elements of Statistics*, p. 35.

must be issued, especially for the non-mathematician. He must bear the above rules in mind and remember, for instance, that the throws "in which we get a six with either of two dice are not so common as the throws in which we get either a six or an ace with one die. We turn up as many sixes with the two dice as we turn up sixes and aces with one; but since the two sixes are on different dice and are therefore not incompatible, they come together in one throw out of thirty-six, and we do not turn them up in so many separate throws."[1] Another point is of interest. The past run of throws can offer no basis for an inference as to the character of the next throw. "To expect that, because a coin has come up heads several times in succession, it is therefore more likely to come up tails the next time, is wholly to misunderstand the meaning of probability. Indeed, a preponderance of heads in the past throws would suggest that the coin was not true, that there was a hidden cause favoring heads, and that as a matter of fact the probability of heads was greater than one half."[2] We see, then, that the calculus of probability is only a guide which we take when we cannot secure definite information about particular cases. Moreover, it is only when certain conditions are fulfilled that the probability in favor of a certain occurrence can be estimated with any large degree of mathematical accuracy.

[1] Aikins, *The Principles of Logic*, p. 340.
[2] Jones, *Logic Inductive and Deductive*, p. 223.

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CHAPTER XXII

AVERAGES AND GRAPHS

The Uses of Averages. In the two preceding chapters we have had frequent occasion to speak of averages. Statistics does not concern itself with the individual case, but with the average for a group or collection. Thus, we are presented with the average income *per capita*, the average number of rooms in the houses occupied by working people, the typical expenditure of a family for food, etc. Insurance, industrial, and health statistics all take this form. It behooves us, therefore, to have some knowledge of the methods used in obtaining such averages and the ways in which they are best presented to the mind.

The following uses summarize very well the view of the modern investigator: —

"Averages are used: 1. *To give a concise picture of a large group.* We could not grasp the idea well if given the height of every tree in a forest, but the average height is something definite and comprehensible.

"2. *To compare different groups by means of these simple pictures.* Thus, two forests can only be compared by means of totals or averages of some sort.

"3. *To obtain a picture of a complete group by the use of sample data only.* An average obtained from a few hundred samples is so close to the exact average of the whole that the difference is negligible.

"4. *To give a mathematical concept to the relationship between different groups.* We may say that the trees in one forest are taller than in another, but in order to find any definite ratio of heights, it is necessary to resort to averages."[1]

The logic of such a situation is clear. In statistics, we stress the quantitative or measurable side of things; and we take them, not as individuals, but as groups. The only way to institute a comparison is to reduce them to some common unit. What this common unit will be depends largely upon the purpose. We shall briefly describe the common averages used and point out their special value.

The Arithmetical Average. The arithmetical average is the sum of the particular items divided by their number. By means of such an average, we are able to handle all the items as a single typical unit which we are able to compare with other like units. Thus, two football teams may be compared in regard to their weight by this means. Suppose the weights of the members of one team were 162, 180, 190, 155, 170, 176, 182, 167, 205, 169, 158 pounds; then the average weight would be 174 pounds. The same method would be used to determine the average for the other team, and so a comparison could be instituted.

It is obvious that such information as the above has its limitations. It makes a deal of difference *where* the weight in a team is distributed, and a general average for the whole group does not bring this out. Hence smaller groups are taken, such as the backfield or the line, and further comparisons instituted. The advan-

[1] King, *Elements of Statistics*, p. 121.

tages and disadvantages of the arithmetical average will be discussed later.

The 'Weighted' Average. "By a 'weighted' average, we mean one whose constituent items have been multiplied by certain weights before being added, the sum thus obtained being divided by the sum of the weights instead of by the number of items." There are two types of weighted averages: one in which groups with varying numbers of members are already averaged and we desire to secure the arithmetical average for the groups taken together; and the other in which the weights stand for estimates of relative importance.

An example will make the first type clear. If, for example, a department store had seven departments and the average wage for each department was known, to obtain the weighted average it would be necessary to multiply the average wage for each department by the number of workers in it, add the results together, and divide by the total number of workers.

The second type appears in the use of 'index-numbers' which represent the estimated importance of the various groups. The following discussion by Bowley brings out very well the nature of the problem and the way in which it is met: —

The classical and most useful application of weights is the formation of an index-number for the change of prices by fitting suitable weights to the changes measured in the prices of various commodities. It is required to find the change in the value of gold when measured by the prices of other commodities. Suppose that we are given that prices of certain commodities between two years were in the following ratios: —

	Wheat	*Silver*	*Meat*	*Sugar*	*Cotton*
First year..............	100	100	100	100	100
Second year............	77	60	90	40	85

The simplest way to estimate for the general fall in price is to take the simple average of the numbers in the second year, viz., 70.4, and say that the general prices in the second year were 70.4 : 100 when expressed in commodities. But it is at once clear that we cannot allow the commodities given to have equal influences on the result; wheat is of greater importance than sugar and meat than silver; and again, we have taken arbitrarily three items to represent food and one for clothing; we need some means of deciding relative importance. Suppose we decide that wheat, cotton, meat, and sugar are respectively 7, 4, 3 times and twice as important as silver, we should get the following table : —

Commodity	*Relative prices in second year*	*Weight assigned*	*Product*
Wheat............	77	7	539
Silver............	60	1	60
Meat............	90	3	270
Sugar............	40	2	80
Cotton............	85	4	340
	352	17	1289

$$\text{Weighted average is} \ldots \frac{1289}{17} = 75.8$$

$$\text{Unweighted average is} \ldots \frac{352}{5} = 70.4$$ [1]

Weights are expressions of expert judgment and are to a certain extent relative. The logician must recognize that a possibility of error lies in their assignment.

[1] Bowley, *Elements of Statistics*, pp. 111–12, quoted from Jones.

Fortunately, it can be demonstrated that an error in weights is less serious in its effects than a mistake in the size of the original items.

The Mode. Another average which is very useful is the mode. It is variously defined as the most frequent size of item, the positions of greatest density, and the quantity which occurs with the greatest frequency. It is what we usually have in mind when we speak of 'the average man,' 'the average income,' 'the average sized apple,' etc. We may say that the mode represents the type.

The mode is not always easily determinable. When the case is simple, the mode can be determined by an examination of a frequency table. The following example should make the method clear: —

Frequency Table showing Heights of Cornstalks

Height in ft. (size of item)	*No. of stalks (frequency)*
m	f
3–4	3
4–5	7
5–6	22
6–7	60
7–8	85
8–9	32
9–10	8
	$n = 217$

It will be noticed that size 7-8 is found 85 times, which is the greatest frequency, and is therefore the mode. There may be two or more frequencies with about equal claims to be the most usual occurrence. Such is the case in the variations of animal forms where one type shades continuously over into another. The

finer the grades, the easier it is to determine the dominant mode.

The value of the mode comes out in the following examples: "The mode rather than the average in chest-measurements is the number most suitable for the ready-made clothier. For providing a post-office or a store, the mode in postal-orders or prices of tea needs to be known rather than any other average. Even the favorite coin in a collection may show the spirit of the congregation better than the arithmetic average of their contributions."

The Median. The median is the middle quantity in a series. "If a number of similar objects are placed side by side in order of their size, they are said to be arrayed. If any group of objects is thus arrayed, the middle one is known as the median item."

The median is of particular value when the data dealt with are not susceptible of measurement in definite units. Psychological phenomena come in this class of material. "It is impossible to measure in specific units the mental characteristics of a child but it is perfectly possible to array a group of children according to their respective mentality."[1] For this reason, the median was used by Francis Galton in his investigations in inheritance and is being used in fields where quality can be graded in a series.

The Geometric Average. The geometric average is obtained by multiplication of the n items of a series, and the extraction of the nth root of the produce. This average soon reaches a complexity which makes it impossible of easy application for those not familiar with

[1] King, *Elements of Statistics*, p. 131.

mathematics. Beyond the square and cube root, it involves the use of logarithms. It is very little used at the present.

The Comparative Advantages of these Averages. The mode is of advantage where we are not concerned with extreme variations which are few in number, but with the type. Its disadvantage is that it is not always easy to determine. The median is easily located and can be determined for all data which can be arrayed in a series. Its chief advantage is that it is applicable to relatively qualitative fields. The arithmetical average is readily calculated and gives weight to all the items; its chief disadvantage is that it requires the presence of all the items and even emphasizes the extremes. The weighted average is a correction of the arithmetical when the series is made up of groups; its chief importance, however, lies in its adaptation to the use of index-numbers. The average used in any concrete investigation is determined by the character of the problem and the nature of the material. Often more than one average is calculated.

Graphs and Graphical Methods. Diagrams are often employed to summarize tables in a picturesque and striking way. Various devices have been worked out to render the meaning of masses of figures clear and unambiguous. The following simple diagrams are often used in the exposition of statistical data: —

1. *Cartograms.* Phenomena with a geographical location can be brought clearly before the eye by means of maps with devices to show variations. Colored printing is sometimes used, but, since this is expensive, various modes of barring are employed. The rainfall maps

printed in the newspapers are good examples of this class. Another method is the use of dots to indicate the relative amount of the product under investigation, each dot standing for a given quantity. Such things as wheat and corn production can be represented in this way so that the results appeal directly to the eye.

2. *Pictograms.* Pictures of comparative sizes, bar diagrams, block pictograms, and circles divided into segments are some of the devices used to bring data before the eye. The importance of such pictorial methods for exposition can hardly be overestimated. "After a person has collected data and studied a proposition with great care so that his own mind is made up as to the best solution for the problem, he is apt to feel that his work is about completed. Usually, however, when his own mind is made up, his task is only half done. The larger and more difficult part of the work is to convince the minds of others that the proposed solution is the best one — that all the recommendations are really necessary."[1] One of the advantages of the graphic method is that the facts can be so presented that the reader can make deductions of his own.

Experience has shown that some forms of charts are far superior to others and that faulty arrangements are easily adopted when criticism has not been directed against them. The bar method has many advantages in that it permits an exact estimation of the component parts and the use of colors to distinguish them. A development of the bar method by subdivision bars is shown in Fig. 15.[2] The sector method is widely used in

[1] Brinton, *Graphic Methods for Presenting Facts*, p. 1.

[2] Figures 15, 16, and 17, on pages 259, 260, 261, are taken from Brinton's *Graphic Methods for Presenting Facts*, by permission of the publishers, *The Engineering Magazine*.

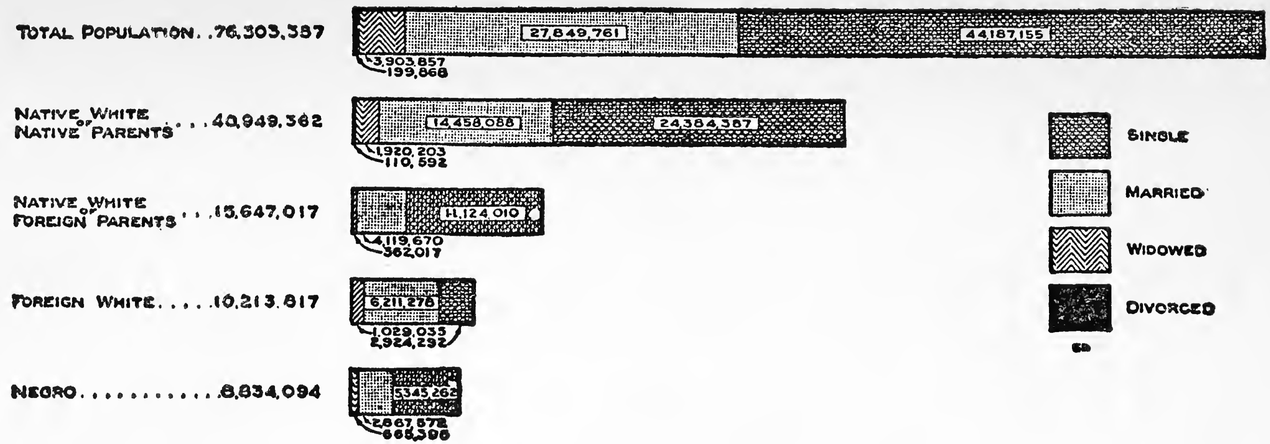

FIG. 15. CONJUGAL CONDITION OF THE POPULATION OF THE UNITED STATES IN 1900

The four lower bars show components of the total population represented by the upper bar. The combined length of the four lower bars equals the length of the upper bar.

popular magazines; its disadvantage lies in its lack of flexibility and the difficulty of arranging and lettering the sectors of the circle. Fig. 16, taken from *The Survey*, is a good example of pictorial presentation.

In almost every chapter we have been compelled to stress the importance of *analysis* for thinking. What is called 'scientific management' is beginning to point out its importance for social and industrial fields. The way in which the graphic method is being employed for this purpose is shown in Fig. 17, taken from *System*.

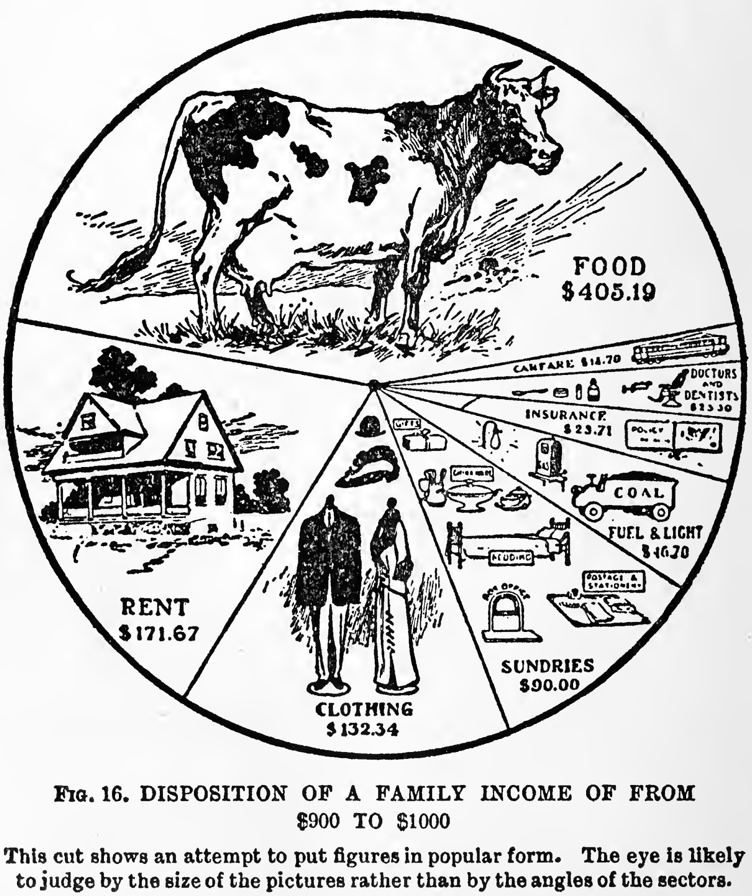

Fig. 16. DISPOSITION OF A FAMILY INCOME OF FROM $900 TO $1000

This cut shows an attempt to put figures in popular form. The eye is likely to judge by the size of the pictures rather than by the angles of the sectors.

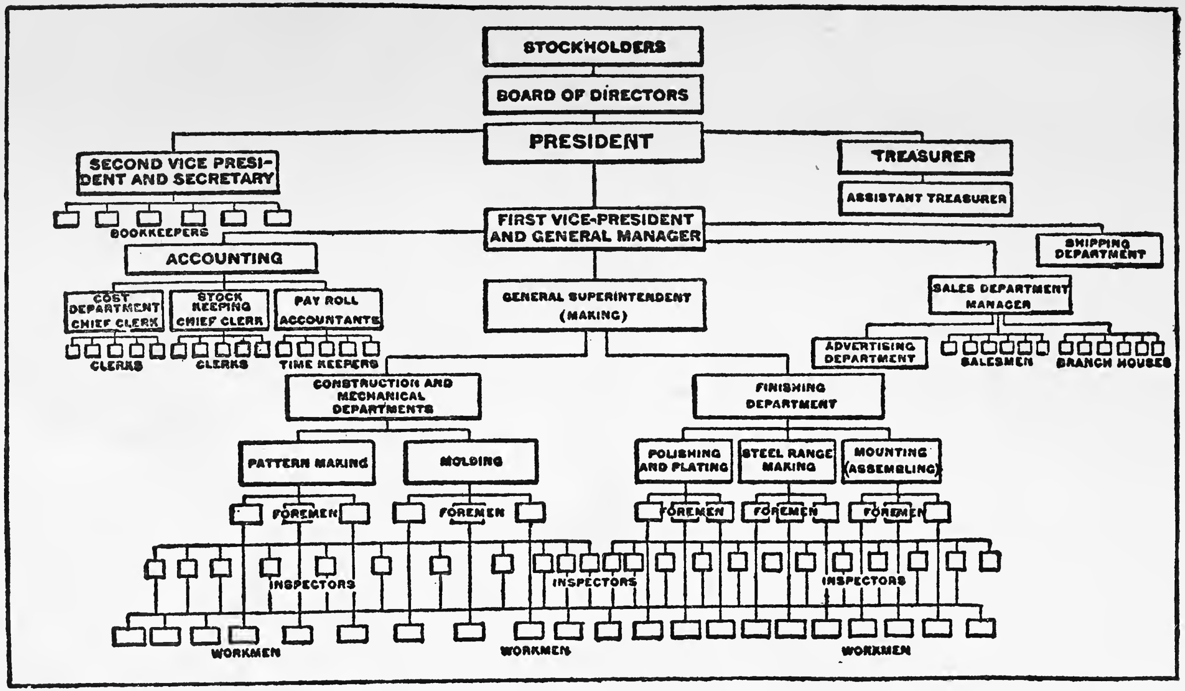

FIG. 17. ORGANIZATION CHART OF A LARGE COMPANY MANUFACTURING STOVES

A complete organization chart should always include the stockholders and the Board of Directors as shown here.

"Authority reaches down through the several branches of an organization like descent of blood, and, if properly planned, it will be as irregular for a factor in an organization to be in doubt as to the person in authority over him as for the child to deny the parentage of his father. . . . It should be graphically shown what positions are only temporarily filled, so that when new men are engaged they will fit into the scheme with functions planned."[1] In such classification, we are dealing with parts of a system rather than with genera and species as in natural science.

3. *Frequency Graphs.* Frequency tables are reduced to graphs in various ways. The foundation is the employment of coördinates cutting each other at right angles at a point called the 'origin.' One set of data is measured along the axis of abscissæ, while the other set is measured along the ordinate. A line drawn through points determined in this way is called a 'graph' and shows the essential points of the table. "For example, if the world's production of wheat over a number of years be plotted, a poor yield is represented by a depression, a rich one by a peak, a uniform one over several years by a horizontal line, and so on." It has been found that graphs dealing with different cases of the same phenomenon can be plotted on the same coördinates and so compared. Often considerable ingenuity is demanded before the graphs assume a form which lends itself to comparison.

[1] Brinton, *Graphic Methods for Presenting Facts,* p. 16.

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CHAPTER XXIII

TESTIMONY AND CIRCUMSTANTIAL EVIDENCE

Scientific Investigation vs. Judicial Proof. Thus far we have largely concerned ourselves with the methods and logical processes of those investigations whose object is the discovery of laws and principles. Such investigations are essentially impersonal and theoretical. They can be carried on in the laboratory and the study, and there is usually less controversy in regard to the *facts* than in regard to the hypotheses to be erected upon them. Many seekers in the same field, having much the same training, accumulate facts by means of critical observation, and these facts are to furnish the foundation of the science. These accepted facts gradually converge in the direction of some satisfactory hypothesis, and so a law of nature or of society is tentatively established. We see, then, that science is coöperative and largely impersonal, its data can be increased by recognized methods until they are sufficient, and it is not limited in the amount of time at its disposal. Nature is besieged rather than taken by storm. How long Newton worked before he published his results is well known, but it is not so well known that Darwin worked upon the development of his theory in regard to the origin of species for more than twenty years.

But there is another great field of systematic reasoning which deserves the attention of the logician. If the methods and temper of the scientist have of late influ-

enced the popular mind in the direction of greater care and exactitude, a similar influence has long been exerted by the methods and principles of *legal reasoning*. Scientific temper is paralleled in great measure by judicial temper. And we must remember that the situation in which legal reasoning is carried on is quite different from that in which scientific investigation takes place, and is much less favorable to good reasoning. The air is full of contention and of the personal element, the time in which to reach a decision is limited, and the necessary facts are often hard to find and verify. Judicial proof is less creative than scientific investigation, but it is in many ways more difficult.

The Difficulties confronting Judicial Proof. The difficulties confronting judicial proof are very similar to those which arise in practical life when the problem is controversial in character. The juror is not in a position to observe the facts, but must reach his decision through the *testimony* of others and through inference from what is called 'circumstantial evidence.' The scientific investigator can repeat his facts at wish, but this the juror, no more than the historian, can do. "The same combination of circumstances which go to make up a case of crime cannot, where they are at all numerous, be expected to occur again. And even if it could or did occur, it would answer no purpose; for it is the identical transaction which took place, and as it took place, which is to be the sole subject of inquiry."[1] The juror's observation is of an indirect, dependent, and therefore inferior kind. Again, the original observer of the facts was seldom aware of their importance. "Many

[1] Burrill, *A Treatise on Circumstantial Evidence*, p. 94.

of the facts . . . not only present themselves to the senses, incidentally, unexpectedly, and transiently, but are, outwardly, and as they present themselves, of the most ordinary and familiar kind, having nothing on their face to attract or arrest attention in any considerable degree; and not to be distinguished from the great mass of facts and events which are constantly passing before the eyes of men, in their daily intercourse with each other. Hence, where they are perceived merely by the organs of sense, without any act on the part of the observer, to give them connection with himself, they are usually perceived in a general and superficial manner." Thus, there is a large amount of possible error in this incomplete and indirect observation upon which, as a foundation, the juror must build his construction of the case. The court must be certain of the identity of the witness and of his general competency, that he was actually an observer of what he testifies to, that his testimony is not biased, etc. Finally, the juror must be able to mass all this evidence, much of which is conflicting, together and draw a conclusion which seems to him to explain the facts of the case. *He is compelled to pass judgment after judgment and then to estimate the comparative value of these integral yet subordinate conclusions in a total view of the problem.* Neither the steps nor the final synthesis is easy of accomplishment in complex cases.

Distinction between Circumstantial and Testimonial Evidence. All evidence involves an inference from some fact to the proposition to be proved. To be evidence, a fact must have a recognizable bearing upon the issue or *probandum.* In such a bearing lies its rel-

evance. Evidence divides itself into two classes, called respectively 'direct' or 'testimonial evidence' and 'circumstantial evidence.' The distinction between these two types is as follows: *Direct or testimonial evidence consists of information derived from assertions made by those who have actual knowledge of the fact; circumstantial evidence consists of facts from which inferences to the principal fact can be drawn.* Thus, testimonial evidence is made up of the assertions of witnesses who are presumed to have first-hand knowledge of the fact at issue, while circumstantial evidence consists of 'circumstances' from which the fact at issue can be inferred according to known laws. It must be remembered, however, that these 'circumstances' must themselves be proved by testimony.

The Nature of Circumstantial Evidence. It is obvious that circumstantial evidence gains its chief importance as supplementary to direct evidence. A common direction for these two kinds of evidence is almost bound to carry conviction. Thus, there may be only one witness who testifies to having seen a crime committed, while many may have seen the accused near the place and showing signs of agitation or marks which fit in with the inference suggested by the prosecution. Thus, testimonial and circumstantial evidence are usually intertwined. But cases occur in which there is no direct evidence, i.e., in which no one has witnessed the commission of the crime. At such times, reliance must be placed wholly upon inference from circumstances. For instance: "A house has been robbed. All the property stolen from it is found directly after the robbery in the possession of *A*. Besides this, at the time when the

robbery is discovered, a hat, usually worn by *A*, is found in the house. It is *almost* certain as a general proposition, that any man of whom these facts are true, must be the robber. And assuming it to be conclusively established that both these facts are true of *A*, we may conclude with an approach to absolute certainty that *A* is guilty of the robbery." When we come to examine all such cases of circumstantial evidence, we find that the principle of explanation is the same as that used in science. *Given certain facts, we must form an hypothesis to explain them.* In a case like the above, the hypothesis that *A* is the robber is the one that naturally occurs. We are certain, at least, that if he is not the robber, he must know who the robber is. *From the standpoint of logic, the proof of the hypothesis consists in the elimination of all alternative possibilities.* The student will recall that Mill's Methods depended largely upon the scientist's ability to exclude possible causes and so narrow down the field.

The Convergence of Evidence. It is seldom that a single circumstance is given such value as to be regarded as conclusive. One instance occurred which should serve to put us on our guard against the easy assumption that there is no other possible hypothesis but the one which most naturally occurs to us. "A man was convicted and executed for stealing a horse, on the strength of the presumption of the animal's being found in his possession on the same day on which it was stolen; but it afterwards appeared that the real thief, being closely pursued by the officers of the law, had met the unfortunate man, to whom he was a total stranger, and requested him to walk his horse for him for a while.

He had then escaped." In practically all cases, then, the prosecution builds up a *framework* of evidence which converges on the hypothesis that the accused is the guilty man. The two facts to be connected are the crime and the individual charged with its commission. The *evidentiary facts* are shown to converge upon such a suggested connection. The figure most frequently employed is that of a chain, but the analogy has a weakness so far as it implies that the disproof of a single link will break the chain. As a matter of fact, the connection is more like a cable woven of strands than a single chain. When each single inference from a circumstance to the issue is probable, the combination of these separate probabilities may lead to a moral certainty through the impossibility of finding an hypothesis, other than the one championed, which will cover them all. The defense then tries to break down the framework of evidence by trying to disprove certain of the alleged facts, by attempting to show that the interpretation of other facts is false, and, in short, by seeking to bring forward another hypothesis as an explanation. And so the struggle between the two sides continues, with the jury as the deciding body.

Direct or Testimonial Evidence. When we are dealing with past events, as in a case of crime and in history, we are compelled to rest our beliefs upon some sort of testimony. Testimony, if true, enables us to reap the results of the perceptions of others, to see indirectly with their eyes. Hence, testimony has an unique value. But it has also its dangers against which the whole procedure of historical investigation and the theory of judicial proof are directed. It is a far cry from the naïve

credulity of the child to the methodical skepticism of the trained historian and the legal expert; yet experience of error has forced this change. "Our first natural attitude toward testimony is one of trust; not because we have reasoned that it is trustworthy, but merely because we cannot help it. If it had not always been natural to accept the statements of those about us as substantially true, we should probably not be alive now to discuss the matter; for the telling of the truth on the one hand and confidence in the story told on the other are very important means for the preservation of the race."[1] But experience soon qualifies this tendency. We discover that we must hear as much evidence as we can, and then compare, sift, and weigh it. Such collection, sifting, and comparing is at first rough and ready. It is soon developed into a careful technique with accepted principles and maxims. The two best examples of such technique are, of course, historical method and legal method. We shall, in the main, concern ourselves with the principles of legal method, but shall now and then call attention to historical method and its manner of dealing with the documents of the past.

The Modern Critical Attitude toward Testimony. "Any assertion, taken as the basis of an inference to the existence of the matter asserted, is testimony, whether made in court or not. Assertions made on the witness stand are merely the commonest class of testimonial evidence."[2] Such assertions are, then, the raw material of proof. They must be tested until a residuum is left as morally certain fact.

[1] Aikins, *The Principles of Logic*, p. 364.
[2] Wigmore, *Principles of Judicial Proof*, p. 312.

When a witness's assertion is examined from the logical standpoint, it is seen to involve at least three elements, *perception, memory, and narration.* In the chapter dealing with "Observation and Fact," we became aware of the ever-present danger of error in both observation and memory. What was said then could be reinforced by case after case in the annals of law.[1]

"A thousand mistakes of every description would be avoided if people did not base their conclusions upon premises furnished by others, take as established fact what is only possibility, or as a constantly recurring incident what has only been observed once. . . . I am assuming that the witness is really desirous of speaking the truth and is merely a bad observer."[2] Much testimony can be proved to be mistaken by fairly simple tests. "Suppose a witness affirms that he was beaten by *H* for ten minutes. Let a watch be placed before him and ask him to take good note of how long ten minutes lasts and then say whether it was really ten minutes. After a quarter of a minute he will exclaim, 'It certainly did not last longer than that.'"

Cross-examination is often of great assistance in determining the validity of assertions. New statements may be made by the witness which will disagree with his other statements, or further developments may be indicated. "In Lincoln's first murder trial, the chief witness had testified to seeing the murder committed by the prisoner. In the cross-examination he added a number of details: that the shooting was at

[1] See Wigmore, *Principles of Judicial Proof*, pp. 296–726.

[2] Hans Gross, *Criminal Investigation*, p. 22.

ten o'clock at night; in beech timber; in August; that he was twenty feet or more away; that he could see the pistol and how it hung; that the nearest lights were half a mile away, and that he saw it all by moonlight. Lincoln showed that the moon did not rise till one o'clock in the morning. Cross-examination may bring out inconsistencies due to dishonesty as well as incompetence, as shown in this example." [1]

Logic has little concern with the practical difficulties due to the temperament of the witness. There are dull and stupid witnesses, timid and self-conscious, hostile, dogged, etc. Logic must, however, stress the part played by suggestion, and the difficulty many people have of expressing themselves adequately. The exhaustive study of testimony in the modern schools of scientific law calls logic, ethics, anthropology experimental psychology, and psychiatry to its aid. Race, age, sex, mental disease, moral character, feeling, interest, and experience, all affect the character of the testimony given.

After the identity of the witness has been established, the first problem is: Was the witness in a position to perceive the facts? The court wishes to know the witness's *opportunities*. Next comes the question whether he has any strong bias or motive for falsifying. Very often the testimony of interested parties is rejected; it is at least somewhat discounted. When the testimony given is against the advantage of the witness, it is, however, regarded as having peculiar weight. Such testimony usually occurs under cross-examination.

[1] Quoted from Jones, *Logic, Inductive and Deductive*, p. 272.

Logical Standards and Tests. There are certain logical tests which are directly applicable to the story told by a witness. *It must be self-consistent; it must be consistent with other known facts of the case; and must be consistent with the larger run of human experience.* Let us look at these three logical tests somewhat closely.

To be acceptable, the testimony of a witness must be coherent and self-consistent. When two of his assertions are shown to conflict, not only are we unable to decide which is the true one, but we are also rightly inclined to be suspicious of the rest of his story.

Again, if the story told conflicts with facts which are otherwise known beyond reasonable doubt, doubt is thereby cast upon it. This is especially the case where the facts are important and conspicuous. The agreement or disagreement of the particular testimony with that advanced by other witnesses enters here as contradiction or convergent confirmation.

Lastly, the statements made must not conflict with the laws of nature or with the tested possibilities of the case. When a man testifies that he saw clearly what occurred in a darkened room when he was looking in from outside, we have good reason to reject his testimony. Of course, assertions must not be too lightly and hastily rejected.

These three tests imply both *internal* and *external* evidence. A very good example from history of their application is the determination of the false authorship of the *Donation of Constantine.* This famous document was the supposed grant by the Emperor Constantine to Pope Silvester and his successors, not only of

spiritual supremacy over the other great patriarchates and over all matters of faith and worship, but also of temporal dominion over Rome and Italy and the provinces, places, and *civitates* of the western regions. A close study of the document, including the style, language, and references, has convinced scholars, both Catholic and Protestant, that the document is a forgery dating from the eighth century. This painstaking application of internal and external evidence has been applied to the Old and New Testament, and is usually called the 'Higher Criticism.' It is very seldom that the alleged author turns out to be the real author.

The Movement of Judicial Proof. Having examined the nature of both circumstantial and testimonial evidence, it may be well to glance at a summary statement of the actual movement of evidence and argument. We shall refer to the asserter of the fact as the 'Proponent' and the opposer as the 'Opponent.' In the following outline, the sign ——> signifies 'tends to prove'; the sign ——o——> signifies 'tends to disprove'; the sign $<$ signifies 'explains away'; letter T stands for a testimonial evidential fact; letter C stands for a circumstantial evidential fact.

Circumstantial Evidence Process

Probandum: X stabbed Y with a knife at a certain time and place.

Proponent's Evidential Fact — C = Bloody knife was found on X ——> *Probandum.*

Proponent's Evidential Fact explained by Opponent — $C = X$ drew it from the wound after the fray on coming to Y's assistance $<$ Proponent's Evidential Fact.

or, —

Proponent's Evidential Fact denied by Opponent — C = Bloody knife was not found on X —o—> Proponent's Evidential Fact.

and this the opponent may do, either —

(1) by adducing new evidential facts — T = M's assertion that on searching X *no* knife was found.

C = No trace of blood from the knife appeared on X's garments.

(2) or by questioning the inference from the T or C on which Proponent's Evidential Fact itself rested as a *probandum.*

Finally, —

Rival New Facts adduced by Opponent — C = X had no quarrel or other motive to stab Y—o—> *Probandum.*

and T = N a bystander asserts that X did not stab Y—o—> *Probandum.*[1]

The Massing of Mixed Evidence. In any complex case, a huge mass of more or less conflicting evidence is presented to the juror's mind, and he must arrange it in such a way as to weigh it correctly. "Many data, perhaps multifarious, are thrust upon us as tending to produce belief or disbelief. Each of them (by hypothesis) has some probative bearing. Consequently, we should not permit ourselves to reach a conclusion without considering all of them and the relative value of each." What the mind ordinarily does under such cases is to pass back and forth from significant

[1] Adapted from Wigmore, *Principles of Judicial Proof*, p. 26.

fact to significant fact allowing itself to balance one fact against another. The result is a drift in a certain direction. In this way, a series of subordinate judgments is passed, and these prepare the field for more comprehensive judgments of a final sort. The mind then goes back over the material in the light of its tentative decision to see whether all the facts fall into line. If they do, the decision is accepted as final. The fundamental mental necessity is the ability to coördinate the data and weigh them in their relation to one another. Along with this must go openness of mind and willingness to go back and forth over the facts with all the reasonable hypotheses in mind to see how adequately they fit.

In such complex reasoning the strands of argument are so many and so interwoven that fallacies of one kind or another are almost sure to creep in unless the utmost vigilance is exercised. Neglect of other possibilities, false disjunction, false analogy, *argumentum ad populum* are fairly common features of forensic contests. In summing up, however, the fallacy which should be most zealously guarded against is the fallacy of the 'neglected aspect.' It is so easy to assume that all the relevant factors have been taken into account and have been given their due weight; *yet nothing is more common than to over-simplify a problem and to neglect important causes and facts.* How many of us have committed this fallacy of neglected aspect in connection with the European War or in connection with social problems like poverty and drunkenness!

The following newspaper argument against prohibition illustrates this tendency very well: "You destroy

the jobs of people upon whom about 5000 of the population depend; you take the jobs away from about 2000 room-rent or house-rent payers or home-owners; you deprive the city itself of about $300,000 of direct revenue, in the way of excise taxes and property taxes; you depreciate the rental value of about $5,000,000 worth of property in the town." Granted that the assertions are correct, "if the facts which they bring forward were the only ones to be considered, the inference as to prohibition would be inevitable. If, however, the liquor traffic is as pernicious in its influence as its opponents claim it to be, the benefits which result from it are far outweighed by the evil it produces. This aspect of the case is neglected however, and so the argument remains inconclusive."[1]

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[1] Bode, *An Outline of Logic*, p. 196.

CHAPTER XXIV

EXPLANATION AND SYSTEM-FORMATION

The Nature of Explanation. To explain anything is to show that it *follows* from something else already known. A fact is explained when it is interpreted by means of a law or a principle. The specific is explained by the general; and the less general by the more general. In explanation, we move downward from rules to cases, from principles to their exemplifications. We see the temporary and the changing in the light of the more permanent and relatively unchanging.

That all explanation is in its essence deduction[1] has been generally recognized by logicians. We try to relate the given to those generalizations which we have slowly worked out as both rules of action and rules of interpretation. We are not comfortable with *mere* facts which are unordered and unrelated. They challenge us as chaotic and *meaningless* until we can somehow put them together as parts of a permanent whole which can be *conceived.* Thus, in the domain of the physical sciences, a fact is felt to be explained when it is related to the terms of a general principle.

The Sentiment of Rationality. A given fact by itself seems like a shot out of a pistol. It *is,* — our senses are, perhaps, witnesses to its occurrence, — *but it stands in isolation.* So far as this is the case, our

[1] Sigwart, *Logic*, vol. II, p. 417.

minds cannot really grasp it and give it meaning for the simple reason that past experience cannot be brought to bear upon it. It remains, then, a brute sensational event to which our concepts cannot attach themselves. But because our minds try to find some point of attachment, some way of striking up a friendship with the event, they feel baffled and uncomfortable. Thus, the sentiment of rationality, the desire to explain and interpret, is an expression of the very mode of working of human consciousness. When any event or thing is given, our minds seek to find relations between it and other facts already familiar and more or less interpreted. In this way it is assimilated and given a local habitation and a name. "All knowledge, all science, thus aims to grasp the meaning of objects and events, and this process always consists in taking them out of their apparent brute isolation as events, and finding them to be parts of some larger whole, *suggested by them*, which, in turn, *accounts for*, *explains*, *interprets them;* i.e., renders them significant. Suppose that a stone with peculiar markings has been found. What do these scratches mean? So far as the object forces the raising of this question, it is not understood; while so far as the color and form that we see mean to us a stone, the object is understood. It is such peculiar combinations of the understood and the non-understood that provoke thought. If at the end of the inquiry, the markings are decided to mean glacial scratches, obscure and perplexing traits have been translated into meanings already understood: namely, the moving and grinding power of large bodies of ice and the friction thus induced of one rock upon another. Something

already understood in one situation has been transferred and applied to what is strange and perplexing in another, and thereby the latter has become plain and familiar, i.e., understood. This summary illustration discloses that our power to think effectively *depends upon possession of a capital fund of meanings which may be applied when desired.*"[1]

The Rôle of Concepts. Concepts play a tremendously important part in both reasoning and explanation. Let us try to bring out the deductive side of explanation referred to above by stressing the rôle of concepts in experience. Because concepts are accepted organizations of experience, anything related to them is naturalized, so to speak, and becomes, thenceforth, a citizen of the mind. Relation to the concepts gives it its credentials. Thus the foundation of all explanation is the capacity of the mind to generalize and to secure by means of analysis, abstraction and synthetic imagination those general ideas and principles which illuminate and organize experience. Our study of induction has been essentially a tracing of the steps and activities involved in the attainment of such concepts. We have seen how facts are observed and selected at the instance of some problem, how comparisons and analyses are made, how guesses at the causes or principles at work are made by fertile minds, how these guesses are tested and modified and re-tested, how systems of conceptual knowledge slowly arise and are used to give meaning to new facts as well as to the old from which they sprung. Now it is to this property of such conceptual systems to give meaning to new facts that we are call-

[1] Dewey, *How We Think*, pp. 117–18.

ing attention. It is in this property that explanation consists.

It is significant that this explanatory property of concepts can be connected with the syllogism. It will be remembered that the typical syllogism has a universal proposition for its major premise. The minor premise then states a concrete case which can be subsumed under the universal principle, while the conclusion consists of the actual identification of the concrete case with the principle.

> All cases of *M* are cases of *P*;
> *S* is a case of *M*;
> Therefore *S* is a case of *P*.

Both *M* and *P* are abstract characters or universals; they are concepts which have gradually been achieved by the human mind as the result of that sentiment of rationality to which we have referred above. "This is, in fact, a world in which general laws obtain, in which universal propositions *are* true, and in which reasoning is therefore possible." We reason down from such general concepts to facts and thereby explain them.

Concepts are "instruments (1) of identification; (2) of supplementation; and (3) of placing in a system. Suppose a little speck of light hitherto unseen is detected in the heavens. Unless there is a store of meanings to fall back upon as tools of inquiry and reasoning, that speck of light will remain just what it is to the senses — a mere speck of light. For all that it leads to, it might as well be a mere irritation of the optic nerve. Given the stock of meanings acquired in prior experience, this speck of light is *mentally attacked by means of appropriate concepts*. Does it indicate

asteroid, or comet, or a new-forming sun, or a nebula resulting from some cosmic collision or disintegration? Each of these conceptions has its own specific and differentiating characters, which are then sought for by minute and persistent inquiry. As a result, then, the speck is identified, we will say, as a comet. Through a standard meaning, it gets identity and stability of character. Supplementation then takes place. All the known qualities of comets are read into this particular thing, even though they have not been as yet observed. All that astronomers of the past have learned about the paths and structure of comets becomes available capital with which to interpret the speck of light. Finally, this comet-meaning is itself not isolated; it is a related portion of the whole system of astronomic knowledge. Suns, planets, satellites, nebulæ, comets, meteors, star-dust — *all these conceptions have a certain mutuality of reference and interaction*, and when the speck of light is identified as meaning a comet, it is at once adopted as a full member in this vast kingdom of beliefs."[1] A fact which is identified, supplemented in this broad way, and put into a system is explained; it has been removed from its isolation, given relations to other things, and interpreted by general principles.

Proof and Explanation. A proposition is proved when it is shown to follow from accepted premises, while a fact or specific rule is explained when it is shown to be the natural consequence of principles. It is evident that proof and explanation are essentially the same; in both the deductive element dominates. Thus

[1] Dewey, *How We Think*, pp. 126–27.

far we have looked at the problem mainly from the angle of explanation; let us now examine it from the side of proof. Why does the application of the major premise to a case constitute proof? "The answer is that it serves to connect the conclusion with the system of concepts or general principles that have previously been accepted. When one sees that the new suggestion comes under the old principles, the belief that has been developed for the system of knowledge extends to the particular instance. The laws and principles that have been established and accepted are connected with the conclusion that is in doubt, and the doubt disappears. Each doubt that is solved increases the belief in the principle, since it assures its connection with a new fact. It should be added that the process of reference to the *system of knowledge*, not merely justifies the old, but also increases the number of applications of the old. It extends its application, and when the conclusion itself is confirmed in practice, the general principle receives new warrant." [1] This quotation brings out admirably the fact that knowledge consists of the growth of systems of knowledge which must take up new facts into themselves. *Such systems possess mental authority, and proof is the expression of this authority.* Combined with such authority there must exist the mental perception of the inner coherence of system and fact. The system must be capable of assimilating the fact; it is here that the deductive element enters.

Systems are tentative. Systems are no longer regarded as having either their final form or content. There is less dogmatism in all fields to-day than formerly.

[1] Pillsbury, *Essentials of Psychology*, pp. 234–35.

The trial-and-error feature of knowledge is more clearly recognized. We may allow ourselves to speak of systems of knowledge as growths, but we must not be misled by the organic analogy. There are differences as well as similarities between the growth of systems of ideas and the growth of organisms. The difference I would stress lies in this, that the innate or determined factor is less obviously in control in systems of knowledge than in the growth of a particular organism. To put this point concretely, systems of knowledge do not have an heredity. If we desire an analogy, it is far better to compare the development of knowledge in a particular field with the adaptation of a human being to his environment. Habits are necessarily tentative and revocable just as ideas are. Man has found that he must relinquish habits and customs and institutions, much as he is forced at times to give up ideas once fondly cherished. In both domains, however, we have good reason to believe that it is not a case of mere supplanting, but that there is a genuine advance, a real progress. I cannot do better in this connection than to quote what Bertrand Russell writes in regard to the attitude and contributions of Henri Poincaré to modern science: "Another reason which makes a philosophy of science specially useful at the present time is the revolutionary progress, the sweeping-away of what had seemed fixed landmarks, which has so far characterized this century, especially in physics. The conception of the 'working hypothesis,' provisional, approximate, and merely useful, has more and more pushed aside the comfortable eighteenth-century conception of 'laws of nature.' Even the Newtonian dy-

namics, which for over two hundred years had seemed to embody a definite conquest, must now be regarded as doubtful, and as probably only a first rough sketch of the ways of matter. And thus, in virtue of the very rapidity of our progress, a new theory of knowledge has to be sought, more tentative and more modest than that of more confident but less successful generations." [1]

Levels of Explanation. We may distinguish different levels in explanations. The first stage is classification. The dominant purpose at this level is to gather objects, compare them, and, finally, classify them. When systems of classification are worked out, specimens can be ordered in relation to one another and new cases can be identified and put in their proper place. No matter how external and superficial such systems of classification are, they represent the beginning of scientific explanation; things are removed from their isolation. As the system of classification is penetrated by perception of more essential characteristics, this stage passes to the next, which may be called 'the empirical.'

At the empirical level, subordinate laws and principles are discovered. For example, many strands of causal uniformity and laws of action and reaction become known. In medicine, it is discovered that certain drugs have definite effects upon the organism, and that the individual reacts in certain ways to various toxins. In chemistry, laws and principles are often known while there is doubt of their real significance. One of the best historical instances of the empirical stage is the situation in astronomy after Kepler and before Newton. Kepler discovered that the planetary orbits are

[1] Russell, *Preface* to Poincaré's *Science and Method*, pp. 6–7.

elliptical instead of circular as had been supposed. Kepler generalized the known facts in terms of his three laws: (1) that the planets move in ellipses round the sun, with the sun in one of the foci; (2) that they describe equal areas in equal times; (3) that the cubes of their mean distances vary as the squares of their periodic times. Although many suggestions were made and analysis of motion was carried further by various physicists, it remained to Newton to carry astronomy over to the third level, that of explanation proper.

Newton did two things: "He conceived that the force which deflected the planets into their orbits was the same as that which made bodies fall to the earth; or, to put it differently, he identified celestial attraction with terrestrial gravity, and conceived the earth as essentially *falling* out of a straight path towards the sun, and the moon towards the earth; and he invented a mathematical calculus by which he could work out what were the *theoretical consequences of the principles he assumed*."[1] Examining these two things which Newton accomplished, we see that they were, first, an extension of the idea of gravity to all cases of physical bodies, an act of universalization, and second, the entrance of a distinctly deductive element. *The more deduction increases, the more has the level of explanation been achieved.*

The ideal often held by the physical sciences is well expressed by Helmholtz. In his little book on the *Conservation of Force*, he writes: "So that at last the task of Physics resolves itself into this, to refer phenomena to inalterable attractive and repulsive forces whose in-

[1] Joseph, *An Introduction to Logic*, p. 479.

tensity varies with distance. The solution of this task would at the same time be the *condition of Nature's complete intelligibility*." For Helmholtz, in other words, partial explanations in science must be subsumed under completer, more deductive ones until the ideal he sets forth is reached. This is not the place to discuss this ideal from the philosophical side. It may be of interest to the student, however, to point out that it is a clear statement of the mechanical view of the world.

General Explanation and Specific Explanation. Sometimes the scientist is primarily interested in discovering general laws and then carrying these laws themselves back to still more general principles; at other times, he wishes to explain particular facts. The more concrete the science, the more is there an interest in the particular for its own sake. Thus, history is mainly concerned with events or customs which have been brought about by the interaction of many forces and agencies. *Such events cannot be deduced from elementary and universal principles.* There is not repetition enough, and the factors concerned are too complex and mutable. Where deduction can afford genuine prediction, there must be relative simplicity. It is possible to predict the transit of Venus, the effect of Iceland spar upon a beam of light, and the product of the union of hydrogen and oxygen; but it is impossible to predict what will happen after the present great war. The historian must grope his way to generalizations which are only tendencies; his chief interest must be directed to the understanding of events which have already happened by tracing their known antecedents. But when the physical scientist seeks to understand just

why some specific event happened, he, also, must search out all the relevant circumstances. The main difference is that *time* is a less important factor for him than it is for the historian. In fact, as we pass from the abstract sciences through the biological to the human sciences, time increases in importance. Institutions and events are best interpreted when we know how they arose. Our conclusion is, that sciences differ in their emphases. Some, the more abstract, aim to furnish general principles only and leave the specific and the unique to the concreter sciences; while the more concrete sciences devote much of their attention to particular events which are judged to be important.

Typical Systems of Knowledge. Many logicians have attempted to classify the sciences according to some one principle. Such classifications must, however, be recognized to be relative to the principle adopted. Suppose that we use the *relative proportion of general law and specific fact* as the standard of division. We shall then commence with mathematics, pass to mechanics, thence to chemistry; from chemistry we shall go to biology and psychology, and, finally, to the human sciences which all involve the methods of history. Corresponding to such a passage from the more abstract to the more concrete is a change in the nature of the material studied. We begin with the abstract properties of physical things and end with highly organized and changing wholes. Let us glance briefly at the two extremes to see what the respective systems are like and how they are developed.

Mathematics is a typical deductive science. It does not involve an appeal to particular facts which must be

generalized. Geometry, for example, does not begin with concrete things as do the physical sciences, but with certain concepts, given in definitions, axioms, and postulates, and with the nature of abstract space. "It is because space relations are unaffected by locality that what I have seen to be a property of this circle must be a property of any circle."[1] What mathematics rests on, then, is an apprehension of the relations between the elements of space and quantity. In a very real sense, there are no particular instances which have relevant qualities of their own. *Thus, the data of mathematics are few in number and essentially changeless, which is another way of saying that they are conceptual rather than perceptual.* The generalization-aspect of the science is taken care of at the very start. It follows that the test of the truth of any conclusion cannot be of a perceptual character; rather is it of the nature of a recognized agreement with the principles and theorems already developed. A geometrical system is primarily a realm of internal consistency. "Thus the space-intuition which is so essential an aid to the study of logic is logically irrelevant: it does not enter into the premises when they are properly stated, nor into any step of the reasoning."[2] The true method of studying geometry is to conceive "interesting simple figures, such as the triangle, the parallelogram, and the circle, and to investigate the correlations between their various parts. The geometer has in his mind not a detached proposition, but a figure with its various parts mutually

[1] Joseph, *An Introduction to Logic*, p. 506. Mr. Joseph's chapter on "Mathematical Reasoning" is well worth study.

[2] Whitehead, *Introduction to Mathematics*, p. 242.

inter-dependent."[1] Such figures are seen in the light of the primary set of axioms and postulates. We may say, then, that mathematics deals with abstract entities which are conceived.

The historical sciences, on the other hand, are founded on concrete data. These data may be classified into two groups, material facts and testimony. *The first duty of the historian is to gather all the data available.* As a rule, such data consist of documents, which are records in various forms of the thoughts and actions of men of former times.[2] No one who has not given some thought to the matter can appreciate the effort required to bring about even a partially satisfactory fund of material, especially where the very distant past is concerned. *The next step is the estimation of the relative value of the various items.* It is to this problem that much of the historian's ingenuity is directed. Both internal and external evidence must be adduced, analyses made, and exact scholarship brought to bear. As material is evaluated, a certain body of more or less certain fact is established and worked up into an interpretative narrative of the development of a people. Such a narrative is being constantly purified by the elimination of data, too credulously accepted, and deepened by the addition of new data and new points of view.

The logician's interest in history concerns the methods used to secure the material, the presence of selection, analysis and interpretation, the part played by guiding-ideas, and the importance of constructive imagination. Modern scientific method in history is a

[1] Whitehead, *Introduction to Mathematics*, p. 238.

[2] See Langlois and Seignobos, *Introduction to the Study of History*.

monument to the ideal of truth in a complex field in which difficulties exist at every step. The historian has been forced by his problems into self-consciousness so that there are several excellent discussions of historical method easily obtainable. Let us look at some of the conclusions which bear upon the elements of induction.

First of all, there have been spirited controversies in regard to the *purpose* of history. There have been political historians, moral historians, sociological historians; and, corresponding to these, there have been historical schools. Upon only one point have all agreed, viz., that truth must be the ideal. Within minor divergencies, *a large guiding-idea is now held in common.* Development is the keynote. "We have noted that the scope of history has broadened by taking more and more factors into account. Wars and statecraft are now regarded as a less important part of national life. This has come about not merely through curiosity to find out how people lived in time past, but in the search after the *explanation of national development.* . . . The thread upon which the story of any nation hangs is development from the past into the present."[1] History seeks to explain, just as any other developed science does, but it explains by development. The composition of causes back of an event or institution is always more or less unique.

The search for documents is a science in itself and involves the collaboration of many subordinate sciences, such as palæography, epigraphy, and philology. But this technique is controlled by the plans and ideas of investigators. Given the material, the next step

[1] Vincent, *Historical Research*, p. 10.

is the determination of facts. "The document is the starting-point, the fact the goal. Between this starting-point and this goal he has to pass through a complicated series of inferences, closely interwoven with each other, in which there are innumerable chances of error; while the least error, whether committed at the beginning, middle, or end of the work, may vitiate all his conclusions."[1] The facts of history, in other words, are judgments, not observations.

There are many sources of error of a distinctly mental character. Sometimes there is reason to doubt the good faith of the author. Sympathy often plays its selective and distorting part; again, vanity and deference to public opinion may be at work. The result of experience has been the rise of a defensive canon, called 'methodical distrust.' This canon will bring home to the student the discussion of some of the causes of error in observation given in Chapter XVII. With a characteristic statement of it we shall close the present discussion of scientific systems, for it generalizes that attitude of reflection and criticism which it is the task of logic on its practical side to enforce. "We must not postpone doubt till it is forced upon us by conflicting statements in documents ; we must *begin* by doubting."[2] We must try to secure "that methodically analytical, distrustful, not too respectful turn of mind which is often mystically called 'the critical sense' but which is nothing else than an unconscious *habit* of criticism."[3]

[1] Langlois and Seignobos, *Introduction to the Study of History*, p. 64.
[2] *Ibid.*, p. 157. [3] *Ibid.*, p. 190.

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CHAPTER XXV

TRUTH AND ITS TESTS

Back to the Definition of Logic. Logic was defined as 'the science of the principles and conditions of correct thinking.' This definition was offered as a guide-post to direct attention to the general character of the subject. Now that we have covered the various topics, we must come back to the definition, analyze the terms, and permit ourselves to suggest and briefly discuss some of the larger and more philosophical problems which have all along hovered in the background. What student has not asked himself Pilate's question. If logic deals with the conditions and principles of correct thinking, it must contain some standard of correctness. While it is not the business of logic to decide *what* ideas are true, — i.e., what the empirical content of truth is, — it does seem to be a part of its task to give analytic knowledge of the meaning of truth and of the principles and methods actually used in testing assertions which lay claim to this apotheosis.

In order to think correctly, we must think both consistently and truly. Let us look at these two prerequisites for a moment. It is impossible to think truly without thinking consistently, but it is possible to think consistently without thinking truly. But very few men want to think *only* consistently. The one is a means to the other, valued largely because of this relation.

The Nature of Consistent Thinking. To think

consistently means to avoid self-contradiction. The conclusions we draw must be related to the premises we accept in accordance with logical necessity. Consistent thinking must not sin against the Laws of Thought and must contain a recognized harmony among its parts. When such thinking is also in agreement with the facts, we are apt to speak of it as true and its product as truth or valid knowledge.

We have come to see that clear and well-organized ideas are essential conditions of consistent thinking. Definition and analysis help to make the ideas we use clear and unambiguous, while classification, in which concepts are ordered in relations of coördination, subordination, and superordination, is another prime condition of good thinking. Only knowledge which is distinct and well organized can be handled without serious danger of error.

Thought discovers internal relations between propositions of such a character that one *follows* from others. The sign of such a logical relation is words like 'therefore' and 'hence.' *M* is *P* and *S* is *M;* therefore *S* is *P*. If we grant the premises, it is impossible to deny the conclusion without self-contradiction. If the dampness of the sidewalk is the sign of either rain or dew, this case of dampness signifies that it rained last night or that there was a fall of dew. The mind feels itself forced to draw the conclusion, once it has admitted a general rule and recognized a specific instance as coming under the rule. Such logical necessity is, in a sense, the inverse of the act of generalization. Were we unable to generalize, we should be unable to infer, and the word, therefore, would have no meaning for us. This

fact reveals the intimate relation of induction and deduction.

The Laws of Thought. We have postponed the treatment of the traditional Laws or Axioms of Thought, because they are apt to appear trivial and meaningless when taken up at the beginning without a logical context. These primary axioms of thought are *three* in number and state the recognized conditions of consistent thinking. So far as I can see, they arise from and express the very nature of conceptual thinking. For that reason, however, they are as self-evident as the axioms of Euclidian geometry. After formulating them, I shall try to relate them to the act of thinking.

The Laws of Thought are: —

(1) The Law of Identity. *A* is *A*.

(2) The Law of Contradiction. *A* is not not-*A*.

(3) The Law of Excluded Middle. Everything is either *A* or not-*A*.

It is obvious that these axioms need some interpretation in order to give meaning to the symbolic form in which they are stated. We shall start with the Law of Identity, develop its meaning, and then show that the other laws are simply further explications of it in the light of the negative.

The Law of Identity carries us back to the nature of concepts. A concept is a mental object which we can hold before the attention, think about, and distinguish from other objects. But before this can be done, these mental objects must have a certain degree of stability and distinctness of content. Hazy, vague, uncertain thoughts are condemned by logic because they are not full-fledged ideas. We all know when we have such im-

mature concepts before our minds — at least, we do when we are reflective and have learned to distinguish between concepts and words. Since one of the principles of logic is non-ambiguity, one of its ideals must be clear and distinct concepts. *Now, such clear and distinct concepts are recognizable and distinguishable from others.* It is this fact that the Law of Identity points out. No thought and no judgment is possible where concepts cannot be held before the mind, retained, reproduced, and recognized as the same. This *sense of sameness* is, as James phrased it, "the very keel and backbone of thinking."

The Law of Contradiction is the negative side of the Law of Identity. If we are able to present clear ideas to the mind, we must also be able to distinguish them from one another. To apprehend *A* is also to know that it is not not-*A*.

There is another form of the Law of Contradiction which concerns itself with judgment rather than with the character of concepts. This other form is usually called the Aristotelian and is as follows: "It is impossible that the same predicate can both belong, and not belong, to the same subject, at the same time, and in the same sense." This formulation brings out sharply the nature and meaning of contradiction by calling attention to the fact that the human mind cannot accept the judgments '*A* is *B*,' and '*A* is not *B*' at the same time.

The Law of Excluded Middle is a further definition of the relation between a term and its contradictory. It asserts that of two contradictory assertions *one* is necessarily true. There can be no third possibility. In formal logic, this principle appears in the fact that

there are only two *qualities*, the affirmative and the negative.

The Postulates of Logic. Besides these axioms, certain postulates are sometimes advanced as essential to logic. The following is a partial list: (1) Every assertion is either true or false. (2) Some propositions may be recognized as true. (3) The mind is able to apprehend objects. (4) There are universal connections in nature, and some at least of these can be discovered. (5) The world is essentially the same for all observers. (6) Individuals who communicate can mean essentially the same by their terms. The investigation of these postulates is usually assigned to epistemology and metaphysics.

The Question of Truth. Consistency with other propositions is not by itself a sufficient criterion of truth. An assertion may follow with due logical necessity from temporarily accepted premises, but such an internal relation cannot guarantee the system as a whole. While the accepted premises justify the conclusion, they themselves must be justified. But does not this situation lead us into a difficulty? Can we have anything more than consistency? Suppose that we use the term 'valid' for a conclusion which follows from accepted premises with logical necessity. The question which confronts us is this, Can we attain anything but *valid* assertions? But, if this be the case, our ultimate premises must either be given with their credentials by some sort of extra-logical intuition, or must be accepted on authority, or, finally, be postulated. Is there any way to avoid this unwelcome series of disjunctions?

I hope that the student has seen that this way of

approach to the question of truth has stressed the deductive side of logic to the exclusion of induction. Were thinking only deductive, we should be compelled to face the unwelcome alternatives outlined above. It has been the prime fallacy of many of the philosophical systems of the past to seek some one principle from which to deduce reality as experienced. In my opinion, such an attempt is illogical because it exalts deduction over induction and does not see that knowledge is a growth in which both aspects, or phases, of thinking are at work.

'Validity' is, then, essentially a deductive term while 'truth' corresponds more to that intimate interplay of induction and deduction which has been presented in our study of science. It will be remembered that there are three elements, or distinguishable mental processes in all systematic investigation, namely, the inductive element *par excellence*, whose principle is observation with its fidelity to fact; the generalizing and hypothesis-forming element; and, lastly, deductive reasoning out of implications. These elements are intertwined according to the exigencies of the case. It is out of the inductive-deductive interaction of fact and theory that truth, as distinct from mere validity, arises.

The Criteria of Truth. It has been customary to speak of truth and to contrast it with error. This collective way of speaking about truth as a body of doctrine has too often led people to think of truth in a mystical fashion. Even critical thinkers have committed the 'fallacy of abstract terms,'[1] and thought of truth as an abstract entity. In order to avoid this dan-

[1] See chapter III.

ger, it is best to distinguish between true ideas and false ideas. And, by ideas, we shall mean assertions, propositions, judgments, theories, in fact all mental objects in regard to which we make the truth-claim. Our present purpose is to discover the actual, empirical tests which are used in investigation to decide between ideas which tentatively claim truth and yet conflict. *What are the criteria of true ideas? And what is the logical setting of these criteria?*

Reviewing our study of systematic investigation, we note that investigation is begun only when some problem arises. Every such problem must have a concrete and specific character. In truth, the scientist is well aware that his first task is to define his problem, for any lack of clearness in the statement of the problem is certain to thwart his efforts. To work on an ill-defined problem is like hunting for something of which one has no clear picture. The next step is the separation of what is certain from what is uncertain. He wants to know what he can depend on, what is beyond question for the purpose in hand. The logician calls the certainties of the field relevant to the specific problem the 'data,' and contrasts the conjectures and hypotheses, which arise in the fertile mind of the scientist, with these data as 'theory.' It must be remembered that such theories develop in the mind of the expert who has detailed familiarity with what has been discovered and achieved in this particular field and in those adjoining. An hypothesis is not external to the knowledge of the field but is a genuine branch of it. The tree of human knowledge is vigorous and puts forth many shoots, some of which must be lopped off. The process

of selecting among competing explanations by marking their capacity to cover and organize the facts and so solve the specific problem is called verification. Verification involves responsibility to fact, freedom from self-contradiction, and a flexible harmony with other accepted theories which have passed through the same test. In any particular case, such testing is a complex process in which the mind works back and forth between data and theory, adding new data and modifying the theory. The criteria of truth are not external but internal. It is absurd to look for some touchstone which can be applied in a mechanical fashion to propositions claiming truth.

Degrees of Belief. Assertions are not made with the same degree of assurance, nor, to state the same fact in more psychological language, is there only one degree of belief. The traditional logic was accustomed to recognize this fact in its theory of *modals*. 'The earth revolves around the sun' is an *assertoric* judgment. A *dictum* is asserted flatly. 'It *may* rain this afternoon' is a *problematic* judgment. In science, the stage of conjecture represents the problematic mode. 'Gravitation may be explained in terms of electricity,' 'Mass may turn out to be a dynamic effect of something more ultimate,' are examples. An *apodeictic* judgment, on the other hand, expresses a sense of logical necessity. The grounds for the judgment are given or intimated. 'There *must* be a presidential election this year' is apodeictic. The ground intimated is, of course, the federal law on the subject.

Modality reflects the fact that ideas have different settings. Beliefs are held with more or less faith. The

upper limit of belief is dogmatic certainty, while the other extreme is a low degree of probability. The logician encourages individuals to give up dogmatic certainty and to make *practical certainty* the highest degree of belief. There is a continuous shading of belief from mere probability, in which the evidence is weak, to practical certainty in which there is no longer any factually motivated doubt. Rational belief is belief founded on grounds, and such grounds must ultimately come back to critically tested fact.

What the Attainment of True Ideas implies. There are at least four things implied in the attainment of true ideas: (1) the ability to obtain facts (observations, testimony, evidence); (2) the right to make use of past experience which is not challenged by reasonable doubt; (3) the Principle of Universal Connections, the Uniformity of Nature, the Law of Universal Causation; (4) the mental capacity for analysis, conjecture, construction, and deductive reasoning.

These four things are largely self-explanatory when taken in the light of what has already been said about them in the preceding chapters. Unless we could obtain facts, which are assertions about which there is no motivated doubt, it would be impossible to generalize, to form hypotheses, or to verify by the convergence of evidence. Unless past knowledge were applicable, it would be impossible to have guidance in an attack on new problems or to determine what was relevant and what irrelevant; again, it would be impossible to build up systems of knowledge such as are found in the various sciences as well as in practical life. Knowledge is, as we have so often pointed out, a growth. While the

old may be modified by the new facts and theories, it yet remains as something substantial within which to work. That universal connections exist and can be discovered is a postulate which underlies all scientific investigation. It cannot be proved in any demonstrative way, but is suggested and in a way verified by experience. Finally, logic assumes that the mental capacities which human beings possess, working upon the material given by observation and testimony, are able to achieve ideas which give genuine knowledge. To doubt this ability to attain knowledge is for logic an ultimate scepticism which it refuses to acknowledge. Both our practical and our intellectual instincts revolt against the suggestion that what we take to be knowledge is not knowledge.

The Meaning of Truth. Having given the empirical criteria of 'truth' and their logical setting, let us be bold enough to discuss the meaning of this term. What do we mean by 'true' ideas? How do such ideas differ from 'false' ideas? Controversial battles have been waged in philosophy over the meaning of these terms. Into the details of such controversies, it is not here the place to enter. We must be brief and clear, and trust that the position adopted will commend itself to the reason of those who have carefully studied the preceding pages.

Let us admit the hazard that what we take to be true may not be true. The problem still remains that we mean something by this term. *An idea is true which meets the empirical criteria and which we therefore take to be a case of knowledge.* An idea is true which has stood the test of motivated doubt, has come out

victorious, and, due to that fact, is accepted by the mind as fulfilling its claim to be an idea of, or about, something. *It is the very nature of an idea to claim to give knowledge.* When this claim is granted by the mind, the idea is accredited or accepted as genuine knowledge; and the term true is used to express this attitude of approval. Thus, it is a contrast term with 'false' as its counterpart. Just as actions are qualified as right or wrong, so ideas — i.e., judgments, propositions, assertions — are qualified as true or false. This latter qualification gets its meaning, however, from the claim to be cases of knowledge which it is the very nature of such judgments to assert.

There can be little doubt that the methods by which ideas are tested enter into the meaning of truth to a greater or lesser extent. What is called 'pragmatism' is a stimulating but, I believe, one-sided emphasis on this aspect of the meaning of truth. "*True ideas*," writes James, "*are those that we can assimilate, validate, corroborate and verify. False ideas are those that we cannot.* That is the practical difference it makes to us to have true ideas; that, therefore, is the meaning of truth, for it is all that truth is known-as." [1] It will be noticed that the meaning of being an actual case of knowledge, which is a part of the content and setting of every assertion, is omitted. This aspect is, in our opinion, fundamental. The logician who understands his science does not deny the empirical character of the criteria of trueness, but he does assert that the claim of an idea to be knowledge is also essential. "Knowledge is an achievement and possession of minds as these

[1] James, *Pragmatism*, p. 201.

have evolved under the stimulus of their environment. As a meaning, knowledge precedes truth, which is a reflective deepening of the sense of knowledge in the light of an awakened doubt. The criteria of truth are, therefore, the same as those of knowledge. Thus truth is accepted and tested knowledge. To say that an idea is true is to say that it is actually a case of knowledge as it claims to be. Truth is knowledge triumphant instead of knowledge militant; yet it is knowledge, as can be seen when we combine the two terms and speak of true knowledge." [1]

Truth and the Will to believe. Of recent years there have been several stimulating essays on the relation of will and our passional nature in general to belief. By consensus of opinion, the two most striking of these are James's *The Will to Believe* and Clifford's *The Ethics of Belief*. These two writers hold positions which are different both in their tenor and in their conclusions. It will repay us to examine their discussions of the foundations of belief for this will introduce us to the larger setting of logic.

Let us begin with *The Will to Believe.* As a psychologist, James rightly first considers the actual psychology of human opinion. "When we look at certain facts, it seems as if our passional and volitional nature lay at the root of all our convictions. When we look at others, it seems as if they could do nothing when the intellect had once said its say." [2] Taking up the latter facts first, he writes: "Does it not seem preposterous

[1] Sellars, *Critical Realism*, p. 282.

[2] James, *The Will to Believe;* page references are to *Representative Essays in Modern Thought.*

on the very face of it to talk of our opinions being modifiable at will? Can we, by just willing it, believe that Abraham Lincoln's existence is a myth, and that the portraits of him in *McClure's Magazine* are all of some one else? . . . We can *say* any of these things, but we are absolutely impotent to believe them; and of just such things is the whole fabric of the truths that we do believe in made up, — matters of fact, immediate or remote, as Hume said, and relations between ideas, which are either there or not there for us if we see them so, *and which if not there cannot be put there by any action of our own*" (p. 76). I feel that it is best to quote this side of James's teaching quite fully, since justice has not been done to it. James permits the 'will to believe' only in certain fields and under certain circumstances. "The talk of believing by our volition seems, then, from one point of view, simply silly. From another point of view it is worse than silly; it is vile. When one turns to the magnificent edifice of the physical sciences, and sees how it was reared; what thousands of disinterested moral lives of men lie buried in its mere foundations; what patience and postponement, what choking down of preference, what submission to the icy laws of outer fact are wrought into its very stones and mortar; how absolutely impersonal it stands in its vast augustness, — then how besotted and contemptible seems every little sentimentalist who comes blowing his voluntary smoke wreaths, and pretending to decide things from out of his private dream" (p. 78).

But, as James points out, there is an interaction between our knowledge, our values, our aspirations, and

our prejudices. The mind is an organic whole, and it is impossible to find an intellect separate from the movement of experience. Facts and theories are not independent, for their discovery, selection, relative importance and control, of the *outlook* of the individual; and this outlook is a synthesis of ideas and feelings. That this is the case we have tried to bring out in the study of induction. But the logician urges the individual to allow the outlook to be more a function of investigation, to make the factual side the control-side. The logician sets an ideal which he admits is seldom approached. This control of outlook by 'fact' is the aim which lies back of the inductive principle of fidelity to fact and the internal criteria of truth, the convergence of relevant evidence. Thus logic stresses the avoidance of error, on the one hand, and the positive methods of attaining new truth on the other. Its aim is to make the individual critically reflective toward assertions and beliefs.

Let us now turn to the thesis which James defends: "*Our passional nature not only lawfully may, but must, decide an option between propositions, whenever it is a genuine option that cannot by its nature be decided on intellectual grounds; for to say, under such circumstances, 'Do not decide, but leave the question open,' is itself a passional decision,—just like deciding yes or no — and is attended with the same risk of losing the truth.*" What strikes the logician in this thesis is the assumption that there are problems "which by their very nature cannot be decided on intellectual grounds" (p. 81). It is this assumption that the logician cannot admit if by intellectual grounds are meant factual

grounds. Such problems could not be formulated and defined.

But James at least suggests that other problems than non-rational ones are also to be treated by our passional nature. And here there is an ambiguity in his treatment. As against Clifford, he insists that we do not need to shut our eyes to a possibility of belief just because the problem has not been settled and there is also a possibility of error. With this I agree. There are degrees of belief; and, when evidence can be advanced both for and against a position, neither affirmation nor negation should be dogmatic. I can understand why such a temperament as James's rebelled against the stoicism of Clough, Huxley, and Clifford which sang: —

> It fortifies my soul to know
> That, though I perish, Truth is so.

When there is probability only, probability is by its very nature both for and against. The probability of throwing a six-spot is one sixth and the chances against are five sixths. James has the right to stress the probability for, while Clifford has an equal right to stress the negative side, the motivated doubt.

But there runs through James the suggestion of another test of truth than that worked out by science and logic. It is not prominent, but it is present enough to have led many to interpret his position in accordance with it. *This second standard consists in an appeal to personal feelings and satisfactions as a foundation for truth.* Such feelings are facts, but they are not the only facts, nor are they the least changing and common of facts. To make them the dominant criteria is, therefore, to forsake an objective and thorough inves-

tigation in favor of what experience has proved to be personal and incomplete.

The Logic of Doubt. In contrast to the essay by James, Clifford's treatment savors almost of a justification of doubt. And, from the logician's standpoint, much can be said in favor of such an attitude. Belief, as a rule, takes care of itself, while doubt and open-mindedness are trained habits not easily acquired. Clifford stresses what may be called the 'ethical aspect' of logic; and in his attitude there is much that reminds us of the Puritan and the Stoic. James was a brilliant, tender-minded, broad-minded mystic; Clifford was a rigorous, tough-minded, intellectual idealist.

"A shipowner was about to send to sea an emigrant ship. He knew that she was old, and not overwell built at the first; that she had seen many seas and climes, and often had needed repairs. Doubts had been suggested to him that possibly she was not seaworthy. These doubts preyed upon his mind, and made him unhappy; he thought that perhaps he ought to have her thoroughly overhauled and refitted, even though this should put him to great expense. Before the ship sailed, however, he had succeeded in overcoming these melancholy reflections. . . . He would dismiss from his mind all ungenerous suspicions about the honesty of builders and contractors. In such ways he acquired a sincere and comfortable conviction that his vessel was thoroughly safe and seaworthy . . . ; and he got his insurance money when she went down in midocean and told no tales.

"What shall we say of him? Surely this, that he was verily guilty of the death of those men. It is ad-

mitted that he did sincerely believe in the soundness of his ship; but the sincerity of his conviction can in no wise help him, because *he had no right to believe on such evidence as was before him.* He had acquired his belief not by honestly earning it in patient investigation, but by stifling his doubts."[1]

Clifford's conclusion is: "It is wrong always, everywhere, and for any one, to believe anything upon insufficient evidence" (p. 54). "'But,' says one, 'I am a busy man; I have no time for the long course of study which would be necessary to make me in any degree a competent judge of certain questions, or even able to understand the nature of the arguments.' Then he should have no time to believe" (p. 54). It must have been this last pronouncement which made James call him "that delicious *enfant terrible.*" Let us soften it somewhat by saying that he should have no time for dogmatic belief, for that belief which shuts out evidence when it comes and passes harshly and uncompromisingly into action.

REFERENCES

Bode, *An Outline of Logic*, chap. XV.

James, *The Will to Believe and other Essays in Popular Philosophy.*

James, *Pragmatism*, chap. VI.

Clifford, *The Ethics of Belief.*

Gibson, *The Problem of Logic*, chaps. I and III.

Sellars, *Critical Realism*, chap. X.

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[1] Clifford, *The Ethics of Belief*, *Representative Essays*, pp. 46–47.

QUESTIONS AND EXERCISES

CHAPTER I

1. Analyze the definition of logic given, and compare it with definitions to be found in other texts.
2. Distinguish between a theoretical and an applied science. Give examples of both.
3. What is meant by 'utilitarian' when the term is used in its narrower sense?
4. Point out and explain why logic has a practical value in education.
5. Name the various kinds of logic. Why have these developed?
6. With what other sciences is logic related?
7. How does the purpose of logic differ from that of rhetoric?
8. To what extent do logic and psychology have the same subject-matter?
9. Where does logic find its material? Why is introspection alone not sufficient?
10. Recall an actual case of reasoning and try to analyze it.

CHAPTER II

1. State some of the mental processes to which the term 'thinking' is applied. Give examples to bring out different levels of thinking.
2. Distinguish between 'imagination,' 'memory,' and 'reasoning.' Make two illustrations of each.
3. What are some of the social conditions of thought? Name some of the great periods in history distinguished by mental boldness and energy.
4. Can you name any great men who owed much to some stimulus which awakened their mental curiosity?
5. "Thinking arises out of the need for adjustment." Explain.
6. What processes are preliminary to reasoning? Distinguish between 'perception' and 'conception.'

7. Collect five statements which you would call 'facts.' Can you find any element of inference or theory in them?
8. Take any field in which you are especially interested, and show that your insight has increased with further study.
9. Give at least four cases where your perception of a thing or of an event has been changed by further knowledge.
10. Do you think that the public schools in America stimulate genuine thinking as much as is desirable? Be prepared to support your opinion.

CHAPTER III

1. Distinguish between a 'logical term' and a 'word.'
2. In the following list, point out which terms are concrete and which are abstract: —

virtue	education	solitude	life
American democracy	asparagus	time	will
detachment	consciousness	auto	cavalry
iron	man	paper	fate

3. Make a list of five examples of collective terms.
4. Distinguish between general and singular terms. Make a list of five of each kind.
5. Do you think that the distinction between singular and general can be applied to abstract terms?
6. Explain the fallacy of hypostatization. Give three examples which show at least an approach to this fallacy.
7. What are relative terms? Give five examples.
8. Explain the danger in discussions about 'poverty,' 'educational progress,' 'the growth of democracy.'
9. Distinguish between positive and negative terms. Give three examples of each. Is the form always an index to the meaning?
10. Explain the distinction between 'denotation' and 'connotation.'
11. Make a series of at least six terms arranged in order of increasing denotation. Arrange the same terms in order of increasing connotation.
12. Show that terms which do not have any connotation still have meaning.

CHAPTER IV

1. Why is language such an important condition of thinking?
2. State the logical law of language.
3. What are some of the causes of ambiguity?
4. Give ten examples of univocal words. Are technical terms always univocal?
5. In the following exercises select those terms of which the meaning is most uncertain. State and justify the meaning which you think belongs to the term in its present context: —
   (1) We should live according to Nature.
   (2) All men have natural rights.
   (3) America is a democratic country.
   (4) International law must be binding on all peoples.
6. Make a list of five words which have decidedly changed their meaning during the last hundred years.
7. Distinguish between vagueness and ambiguity. Show why abstract terms are peculiarly liable to vagueness.
8. Try to formulate the divergent notions of 'force,' 'matter,' and 'motion' that are expressed or implied in the following extracts from Buechner's *Force and Matter:* —

   "No force without matter — no matter without force. One is no more possible, and no more imaginable by itself than the other. . . . Force and matter are fundamentally the same thing, contemplated from different standpoints. In the material world we know of no example of a particle of matter not endowed with force or working by it. We must further admit on closer investigation, that matter as such could make no impression on our sense-organs or minds; it can only do this by means of the forces united with or at work within it. A piece of lead held in the hand presses on it because of the attractive force of the earth and so produces the idea of weight. . . . Nothing can prove to us the real existence of a force, except the properties, changes and movements, which we become conscious of in matter, and these we call different 'forces' according to the resemblances or differences in such manifestations; any knowledge

of them by other ways is impossible. . . . Force may be defined as a condition of activity or a motion of matter or of the minutest particles of matter or a capacity thereof; yet more precisely, as an *expression for the reason of a* possible or actual movement. . . .

"Motion must be regarded as an eternal and inseparable property or as a necessary condition of matter. Matter without motion exists no more than matter without force; motion without matter exists as little as force without matter. Nor can motion be deduced from any force, for it is the very essence of force itself, and can therefore have no origin, but must be in all places. . . . The most solid body owes its condition only to the mutual attractive force of its minutest particles, which continually oscillate or swing round the so-called center of gravity, and without which it would at once fall to pieces. That these particles are never able to attain a condition of relative rest is proved by the universally present force of heat, which is known to be nothing more than a mode of motion and which, since all bodies without exception contain heat, keep these smallest particles or molecules in a state of continual movement. . . . Motion must therefore be regarded as the primal condition or in some measure as the soul of matter." [1]

CHAPTER V

1. What is classification? Out of what need does it arise?
2. Distinguish between 'artificial' and 'natural' classifications.
3. How has the theory of evolution affected scientific classification?
4. What is a 'dichotomous' division? Under what conditions might it be advantageous?
5. Criticize the following divisions: —
   (1) Religions, into Christian, Mohammedan, and non-Christian.
   (2) Pictures, into paintings, engravings, posters, and pen-and-ink sketches.

[1] Quoted from Bode, *An Outline of Logic.*

(3) Schools, into technical, vocational, and public.
(4) Furniture, into Colonial, Sheraton, Adams, office, and home.
(5) Political parties, into Republican, Radical, and Conservative.

6. Divide and subdivide the following: —

Governments, Sciences, Sports, Schools.

7. State and explain the technical terms used in division.
8. What part does purpose play in both classification and division?

CHAPTER VI

1. Why is definition needed?
2. Show the relation between 'definition' and 'classification.'
3. Define the following terms by proximate genus and essential difference: —

Democracy, Education, Tariff, Politics, American, Honor, Wealth, Neutrality.

4. Examine the following definitions to see whether they conform to the logical rules of definition: —
   (1) The body is the emblem or visible garment of the soul.
   (2) Life is a continuous adjustment of internal to external relations.
   (3) A phonograph is a mechanism for recording and reproducing sounds.
   (4) Life is the opposite of death.
   (5) Psychology is the science of the processes whereby an individual becomes aware of a world of objects and adjusts his actions accordingly.[1]
   (6) Psychology is the science of the phenomena of consciousness.[2]
   (7) Education is the eternal process of superior adjustment of the physically and mentally developed, free, conscious human being to God, as manifested in the intellectual, emotional, and volitional environment of man.[3]
   (8) Education is conscious or voluntary evolution.[4]

[1] Stout. [2] Baldwin. [3] Horne. [4] Davidson.

(9) Religion consists in the feeling of absolute dependence.[1]
(10) Religion is a desire manifested by prayer, sacrifice, and faith.[2]
(11) Religion is a faculty of the mind by which, independently of the senses and of reason, man is able to perceive the Infinite.[3]

5. Show that a definition is always relative to the purpose entertained.
6. What are the predicables? Distinguish between a 'differentia' and an 'accident.'
7. Write out a list of at least five words or phrases which you regard as catch-words which need definition.
8. Look up the platforms of the various political parties for words which seem to you to need definition.

CHAPTER VII

1. Distinguish between 'assertions' and 'propositions.' What was the error of the more formal logic of the past?
2. Show that judgment is a process of interpretation ending in an assertion.
3. Bring out by at least three examples the genetic side of judgment.
4. Show that the world as we experience it is a product of past mental activity.
5. In what sense are there levels of judgment? Give cases to prove the increasing complexity of judgment as we pass from concrete things to principles.
6. What two aspects are there to every judgment?
7. Distinguish between the actual operation of judging and its verbal expression. What is the purpose which controls language?
8. What is meant by the 'universe of discourse'? Indicate the reason for its importance in the proper interpretation of propositions.

[1] Schleiermacher. [2] Feuerbach. [3] Max Müller.

CHAPTER VIII

1. Distinguish between 'categorical,' 'hypothetical,' and 'disjunctive' propositions.
2. What is meant by the 'quantity' of propositions? By their 'quality'?
3. Throw the following propositions into logical form, give the letter that symbolizes the quantity and quality, and state the distribution of the terms: —
   (1) Not all who are called are chosen.
   (2) Few were saved.
   (3) None of the planets except the earth is inhabited.
   (4) He can't be wrong whose life is in the right.
   (5) More haste less speed.
   (6) All politicians are not dishonest.
   (7) Only noble deeds deserve praise.
   (8) Mercy but murders, pardoning those that kill.
   (9) No one is always happy.
   (10) Only some citizens have the right to vote.
4. Explain why negative propositions always distribute the predicate.
5. Apply the graphical method to the four kinds of categorical propositions.
6. What is the purpose underlying the logical manipulation of propositions.

CHAPTER IX

1. What is immediate inference?
2. Be prepared to write down and explain the Square of Opposition.
3. What propositions are true, false, or doubtful, —
   (1) when *A* is false; (3) when *I* is false;
   (2) when *E* is false; (4) when *O* is false?
4. What is the simplest proposition which must be established in order to disprove the following statements: —
   (1) All men desire wealth.
   (2) No man is perfectly happy.
   (3) Some knowledge is not of any value.
   (4) Pain alone is evil.
   (5) All is not lost.[1]

[1] Creighton.

5. Give the converse, the obverse, and the contrapositive of each of the following propositions: —
   (1) Only a fanatic believes in panaceas.
   (2) Uneasy lies the head that wears a crown.
   (3) No men are always happy.
   (4) Most of the nations were unprepared.
   (5) None but the industrious deserve to succeed.
   (6) All men are mortal.
   (7) Discontent is frequently a symptom of inefficiency.
   (8) Children are noisy on rainy days.
6. By what process do we pass from each of the following propositions to the next?
   (1) No knowledge is useless.
   (2) No useless thing is knowledge.
   (3) All knowledge is not useless.
   (4) All knowledge is useful.
   (5) What is not useful is not knowledge.
   (6) What is useless is not knowledge.
   (7) No knowledge is useless.[1]
7. Test the following arguments by obversion and conversion: —
   (1) When we hear that all the righteous people are happy, it is hard to avoid exclaiming, What! are all the unhappy persons we see to be thought unrighteous?
   (2) If a man who has been accustomed to enjoy liberty cannot be happy in the condition of a slave, does it follow that a man who has not been accustomed to liberty can be happy as a slave?

CHAPTER X

1. How did Aristotle define the syllogism? What was the germ of his invention?
2. What is the *Dictum de omni et nullo?*
3. Be prepared to demonstrate each of the rules of the syllogism.
4. What criticism would you be inclined to pass upon the syllogism when treated very formally?
5. Select and name the various propositions and terms of

[1] Jevons.

the syllogism in the following informally stated arguments. Then reconstruct them so as to make formally correct syllogisms: —

(1) All leeches must be true worms; for all annelids are worms, and leeches are annelids.

(2) Caterpillars have true legs; worms do not; and so caterpillars are not worms.

(3) Amphibians are not reptiles, since they breathe by gills in the larval stage, and reptiles do not.[1]

6. If either premise of a syllogism is *O*, what must the other be?

7. Prove that there must be in the premises one more distributed term than in the conclusion.

8. Prove that if one, but only one, premise is negative, and both premises are universal, they will between them distribute three terms.

9. Put the following argument into syllogistic form: —

How can any one maintain that pain is always an evil, who admits that remorse involves pain, and yet may sometimes be a real good?

CHAPTER XI

1. Arrange the following arguments in logical order. Name the figure of the syllogism, and if the argument is invalid, state the formal fallacy involved: —

(1) All *M* is *P;* no *M* is *S;* therefore no *S* is *P*.

(2) No *P* is *M;* some *S* is *M;* therefore some *S* is not *P*.

(3) Some useful metals are becoming rarer; iron is a useful metal, and is therefore becoming rarer.

(4) None but whites are civilized; the ancient Germans were white; therefore they were civilized.

(5) Since the virtuous alone are happy, he must be virtuous if he is happy, and he must be happy if he is virtuous.

(6) It is not true that a man cannot do a great work without a strong physique; for the philosopher Kant did a great work and his physique was anything but strong.

[1] Taylor.

(7) Only animals are sentient beings; fishes are animals; therefore fishes are sentient beings.

(8) Good men write good books; this is a good book; therefore its writer was a good man.

(9) Every book is liable to error; every book is a human production; therefore all human productions are liable to error.

2. Write out the sixty-four moods of the syllogism and strike out the fifty-three invalid ones.
3. Prepare to prove the special canons of each of the four figures.
4. Make a syllogism in *E I O*, any figure, and exhibit the conversions necessary to accommodate it to each of the other figures.[1]
5. Why is the first figure usually considered the standard figure?
6. Supply premises for the following conclusions: —

(1) Some politicians are not dishonest.

(2) The moon tends to fall to the earth.

(3) Banks sometimes fail.

(4) Some logicians are not good reasoners.

CHAPTER XII

1. Complete the following arguments, determine their mood and figure, and decide whether they are valid: —

(1) Blessed are the meek, for they shall inherit the earth.

(2) Only the good are fit to die, therefore capital punishment is wrong.

(3) Let us meet her. Why? They say she's mad.

(4) Because thou art virtuous, shall there be no more cakes and ale?

(5) None but material bodies gravitate; therefore air is a material body.

2. Construct an enthymeme of each of the three orders.
3. Construct a complex argument containing a prosyllogism and an episyllogism.
4. Show why all the premises except the first must be universal in an Aristotelian Sorites.

[1] Taylor.

5. Prove by graphical methods that in the Goclenian Sorites the first premise alone can be negative, and the last alone particular.
6. What is an extra-syllogistic argument? Contrast it with syllogistic inference.
7. Why are fallacies more apt to exist in condensed arguments than in expansed ones?

CHAPTER XIII

1. Determine which of the following give valid conclusions and which do not; in case of invalidity, name the fallacy:
   (1) If the door were locked, the horse would not be stolen; but the horse is not stolen, therefore the door must have been locked.
   (2) If all men were capable of perfection, some would have attained it; but none having done so, none are capable of it.
   (3) If every ghost story is to be believed, we must accept the general standpoint of the spiritualists; but we cannot accept their general standpoint; therefore we cannot believe ghost stories.[1]
   (4) 'If he has not studied, he will fail in the examination.' With this proposition as a major premise, what can be inferred if we take as minor premise:—
      (1) He has not studied.
      (2) He will fail.
      (3) He will not fail.
      (4) He has studied.
   (5) If it becomes colder to-night, there will be a frost; but it will not become colder to-night; therefore there will be no frost.
2. Does the categorical or the hypothetical syllogism seem to you the simpler? Which of the two stresses denotation?
3. Where is the source of danger in the disjunctive syllogism?
4. Work out a dilemma on some subject of debate.
5. Examine the following arguments: —
   (1) It is either raining or not raining; it is not raining; therefore it is raining.

[1] Aikins.

(2) No honest man can advocate a change in the creed of his church; for he must either believe it or not believe it; and if he believes it, he cannot honestly help to change it, while if he does not believe it he cannot honestly belong to the church at all.

(3) In order to move, a body must move either in the place where it is or in the place where it is not. But it cannot move in the place where it is, since that place is already occupied. Neither can it move in the place where it is not. Motion is therefore impossible.

(4) If transportation is not felt as a severe punishment, it is in itself ill-suited to the prevention of crime; if it is so felt, much of its severity is wasted, from its taking place at too great a distance to affect the feeling, or even come to the knowledge, of most of those whom it is designed to deter; but one or the other of these must be the case; therefore transportation is not calculated to answer the purpose of preventing crime.

CHAPTER XIV

1. Distinguish between a 'sophism' and a 'fallacy.'
2. Review the formal fallacies.
3. Be able to classify material fallacies under two appropriate headings.
4. Look up advertisements to see whether you can find some examples of amphiboly.
5. Give an example of the fallacy of accident. Also of the converse fallacy of accident.
6. Explain the difference between 'begging the question' and 'irrelevant conclusion.'
7. Give an example each of *Argumentum ad hominem*, *Argumentum ad populum*, *Argumentum ad ignorantiam*, and *Argumentum ad vericundiam*.
8. Note and classify the fallacies in the following arguments: —

   (1) A monopoly of the sugar-refining business is beneficial to the sugar-refiners; and of the corn trade to

the corn-growers; and of silk manufacture to the silk-weavers; and of labor to the laborers. Now, all these classes of men make up the whole community. Therefore a system of restrictions upon competition is beneficial to the community.[1]

(2) Any student in college would stand higher in his class if he received higher marks; hence, if all marks were raised ten per cent every man would stand nearer the head of his class.

(3) Since attending that mass meeting of students, I have had no confidence in decisions reached in that way.

(4) Wine is a stimulant; therefore, in every case where a stimulant is harmful, wine is harmful.

(5) We know that God exists, because the Bible tells us so; and we know that whatever the Bible affirms is true, because it is of divine origin.

(6) A miracle is incredible because it contradicts the laws of nature.

(7) We charge him (King Charles the Second) with having broken his coronation oath, and we are told that he kept his marriage vows; we accuse him of having given up his people to the merciless infliction of the most hot-headed and hard-hearted of prelates, and the defense is that he took his little son on his knee and kissed him; we censure him for having violated the articles of the Petition of Rights, after having for a good and valuable consideration promised to observe them, and we are informed that he was accustomed to hear prayers at six o'clock in the morning.

What fallacy does Macaulay refer to in this passage? [2]

(8) If it is fated that you die, you will die whether you call in a doctor or not, and if it is fated that you will recover, you will recover whether you call in a doctor or not. But it must be fated either that you die or that you recover. Therefore, you will either die or recover, whether you call in a doctor or not.

[1] Hyslop. [2] Russell.

(9) Every incident in the narration is probable; hence, the narrative is probable.

(10) The growing size of London bodes evil to England because London is the heart of England, and a swollen heart is a sign of disease.

9. "The President of the United States can veto bills, but he can only veto a bill as a whole. It is therefore not uncommon for Congress to tack on to a bill which the President feels bound to pass a clause containing a measure to which it is known that he objects." Show that this is a practical application of a fallacy.

10. "One of the most remarkable examples of fallacy is furnished by the political theory of Hobbes and Rousseau, known as the theory of the 'social compact.' We are supposed bound by the promise entered into by our ancestors before society was called into existence; but there is no such thing as an obligatory promise until society has been formed." What important assumption is made in this criticism of the compact-theory?

11. "How is it that we hear so much of French immorality, and nothing, or next to nothing, of Italian? How is it that, in France, we have heard so much of English barbarity and cruelty, whilst the accounts of Turkish cruelty were received with the smile of incredulity or the shrug of indifference?" [1] What tendency to fallacy does this quotation illustrate? Give other instances.

12. "Achilles and a tortoise run a race together. Achilles runs ten times quicker than the tortoise, and accepts, in consequence, a handicap of a hundred yards. Under these conditions, argued Zeno, Achilles will never overtake the tortoise; for when the tortoise has gone ten yards, Achilles will still be ten yards behind him. When these ten yards are caught up, the tortoise will still be ahead by one yard. When this yard is caught up, one-tenth of a yard will still separate them, and so on indefinitely. Achilles, then, though he will be continually drawing nearer to the tortoise, will never actually overtake him." How would you criticize this argument?

[1] Hammerton, *French and English.*

FURTHER EXAMPLES OF DEDUCTIVE ARGUMENTS

CHAPTERS X TO XIV

1. The right should be enforced by law; the exercise of the suffrage is a right, and should therefore be enforced by law.
2. Every rule has exceptions; this is a rule, and therefore has exceptions; therefore there are some rules that have no exceptions.
3. Is a stone a body? Yes. Then is not an animal a body? Yes. Are you an animal? I think so. Ergo, you are a stone, being a body.
4. His imbecility of character might have been inferred from his proneness to favorites; for all weak princes have this failing.
5. At the time of the Galveston flood men worked sixteen hours a day; hence, to talk of an eight-hour day as a necessity for the working classes is absurd.[1]
6. Haste makes waste, and waste makes want; therefore a man never loses by delay.
7. A college education does not pay for most self-made Americans have succeeded without it.
8. All material bodies impress the senses; mind does not impress the senses. What is the inference? [2]
9. Art is not fostered by money; for a true artist would practice his art for its own sake, and a bad artist should not be encouraged.
10. Wealth is in proportion to value, value to efforts, efforts to obstacles; therefore wealth is in proportion to obstacles.
11. For those who are bent on cultivating their minds by diligent study, the incitement of academic honors is unnecessary; and it is ineffectual for the idle and such as are indifferent to mental improvement; therefore the incitement of academic honors should be abolished.
12. It is impossible to be a good shot without having a

[1] Jones. [2] Taylor.

steady hand; John has a steady hand; he is capable, therefore, of becoming a good shot.

13. The theory of evolution is not true, for it was not accepted by Agassiz or by Gladstone; moreover, you cannot accept this doctrine, for it is disclaimed by the authorities of your church.[1]
14. The end of a thing is its perfection; death is the end of life; therefore death is the perfection of life.
15. Improbable events happen almost every day; but what happens almost every day is a very probable event; therefore improbable events are very probable events.
16. No evil should be allowed that good may come of it; all punishment is an evil; therefore no punishment should be allowed that good many come of it.
17. Why does a ball, when dropped from the masthead of a ship in full sail, fall, not exactly at the foot of the mast, but nearer to the stern of the vessel?
18. Written examinations are not absolutely fair tests of a student's scholarship — much less of his industry and intelligence. It is therefore wrong to base his grade upon them.
19. A vacuum is impossible, for if there is nothing between two bodies they must be in contact.
20. "They tell us that we are weak, unable to cope with so formidable an adversary; but when shall we be stronger?"
21. Nothing is better than wisdom; dry bread is better than nothing; therefore dry bread is better than wisdom.
22. Whoever believes this is a heretic; so that you are no heretic, for you do not believe this.
23. If a man is educated, he does not want to work with his hands; consequently, if education is universal, industry will cease.
24. "The railroads have usually acted upon the apparent policy that it is none of the public's business whether they are overcapitalized or not. It remained for the counsel for the N. and N. Railroad, a road notorious for its stock-watering operations, publicly to declare — in form of a question, it is true, but none the less bluntly — the railroad position. If the N. and N. 'charges reasonable rates,' demands its counsel, 'what is it to the public

[1] Creighton.

whether its capitalization be high or low?'" What assumption is involved in this question?[1]

25. Berkeley's Theory of the Non-Existence of Matter is palpably absurd, for it is impossible even to place one's foot on the ground without experiencing the resistance of matter.
26. I have no hesitation in saying that the proposition, however good in theory, is in practice utterly absurd.
27. Epimenides the Cretan says that "all Cretans are liars," but Epimenides is himself a Cretan; therefore he is himself a liar. But if he be a liar, what he says is untrue, and consequently the Cretans are veracious; but Epimenides is a Cretan, and therefore what he says is true; hence the Cretans are liars, Epimenides is himself a liar, and what he says is untrue. Thus we may go on alternately proving that Epimenides and the Cretans are truthful and untruthful.
28. There exist many differences of opinion and much uncertainty with regard to many questions connected with geology; consequently geology is not a science, and any arguments which assume the truth of geological theories must invariably be regarded with considerable suspicion.
29. Personal deformity is an affliction of nature; disgrace is not an affliction of nature; personal deformity is not a disgrace.
30. Testimony is a kind of evidence which is very likely to be false; the evidence on which most men believe that there are pyramids in Egypt is testimony; therefore the evidence on which most men believe that there are pyramids in Egypt is very likely to be false.
31. Why should any but professional moralists trouble themselves with the solution of moral difficulties? For, as we resort to a physician in case of any physical disease, so, in the case of any moral doubt or any moral disorganization, it seems natural that we should rely on the judgment of some man especially skilled in the treatment of such subjects.
32. "*A* asserts with incorrigible optimism that 'without too much you cannot have enough of anything. Lots of inferior books, lots of bad statues, lots of dull speeches,

[1] Bode.

of tenth-rate men and women, as a condition of the few precious specimens in either kind being realized.' As the condition, yes; but as the cause, no. We can never have the precious things in literature merely by adding to the multitude of cheap things."[1]

33. "An abundant stream divides two limits of one property . . . and over this stream stood a bridge; and at the head of it a gallows, over which were appointed four judges to decide according to the law established by the lord of the stream, the bridge, and the territory. The law ran in this wise: 'If any one shall pass over this bridge from one side to the other, he must first swear as to whence he comes and on what business he is bound, and if he swear truly, he must be allowed to go; but if he swear falsely, he shall on that account die by hanging on the gallows which is there; and that without remission whatever.' This law and its stern conditions being known, many went over; and as soon as it was perceived that they swore truly, the judges allowed them to pass freely. It happened, however, that on swearing one man, he took the oath and declared that he was going to die on that gallows, and that he had no other business. The judges consulted the terms of the oath, and said: 'If we allow that man to go free, he has sworn falsely, and according to the law he ought to die; and if we hang him, the oath that he was going to hang on that gallows was true, and according to the same law he ought to be free.'"[2]

34. We must either gratify our vicious propensities, or resist them; the former course will involve us in sin and misery; the latter requires self-denial; therefore we must either fall into sin and misery or practice self-denial.

35. "America has still a long vista of years stretching before her in which she will enjoy conditions far more auspicious than any European country can count upon. And that America marks the highest level, not only of material well-being, but of intelligence and happiness, which the race has yet attained, will be the judgment of those who look, not at the favored few for whose benefit the world seems hitherto to have framed its institutions, but at the whole body of the people."[3]

[1] Creighton. [2] From *Don Quixote*. [3] James Bryce.

36. This is a party measure and therefore we must vote for it.
37. "Philosophy bakes no bread." Then why waste time upon it?
38. I oppose this bill because it involves an infringement of the rights of the liberty-loving citizens of this State.

CHAPTER XV

1. Why are 'induction' and 'deduction' somewhat misleading terms?
2. Show how the perspective of logic has shifted from age to age.
3. Distinguish between the 'logic of consistency' and the 'logic of investigation.'
4. Look up an instance of systematic investigation, such as can be found in Darwin, Newton, Faraday, etc., and analyze the various logical steps taken.
5. What are the three elements which can be distinguished in investigation?
6. What is involved in generalization? Exemplify the passage from the particular to the general.
7. What is meant by an 'internal relation'? How does the relation between a sign and that which it signifies differ from association?
8. Do animals generalize?
9. Discuss the statement, assigned to Wundt, that "animals never reason and man seldom."
10. Is the tendency to *test* generalizations as characteristic of man as the tendency to generalize itself?

CHAPTER XVI

1. What do you understand by science? What are some of its guiding ideals?
2. Look up the history of some science with which you are fairly familiar, and ask yourself what its growth consists in.
3. Why is analysis so basic for science?
4. Point out the significance of technique and instruments. Are they, in your opinion, sometimes overestimated at the expense of mental factors?

5. What is the nature of experimentation? Indicate the advantages of experimentation as against passive observation.
6. Find and analyze at least two important experiments in each of the following sciences: physics, chemistry, biology, and psychology.
7. What other methods have grown up in modern investigation?
8. What reaction has science had upon common-sense thinking?
9. Become acquainted with the mental life of at least one great scientist.

CHAPTER XVII

1. What principle should guide induction?
2. What is meant by 'mere speculation,' and why is it condemned?
3. State some of the mental and physical conditions of accurate observation.
4. Show that perception is a process involving associated ideas as well as sensations.
5. What are the causes of mal-observation and non-observation in the following cases?
   (1) A straight stick partly immersed in water seems to be bent.
   (2) The sun seen through a fog sometimes appears red.
   (3) Patients often seem to feel pain in amputated limbs.
   (4) A rearrangement of the furniture in a room is often unnoticed.
   (5) There are marked differences in what the ordinary good observer, the artist, and the botanist see in a flower.
   (6) Silas Marner mistook Effie's hair for the lost gold.
   (7) Looking at one's watch and not knowing the time a moment later.
   (8) Shooting a man for a deer when hunting in the woods.[1]

[1] Jones.

6. Seek for cases of erroneous perception in your own experience, and try to explain them.
7. Is memory a direct intuition of the past?
8. How do the sciences try to eliminate errors due to memory? Why cannot this be done in the courts of law?
9. Present at least five instances of erroneous memory.
10. How is the relevancy of facts determined?

CHAPTER XVIII

1. What is an hypothesis? And what are its conditions?
2. Show by examples that hypotheses always arise in answer to problems.
3. Criticize the Baconian view of science.
4. What do men like Ernst Mach and Wilhelm Ostwald mean by hypotheses when they are inclined to urge scientists to avoid them so far as possible?
5. Take any two instances of hypotheses which have become accepted theories and show that only the expert could have thought of them.
6. Distinguish between the 'intuitive' and the 'reflective' type of mind. Are these types easily recognizable in practice?
7. Name three famous men who represent, on the whole, the intuitive type, and three who are quite clearly of the reflective type.
8. What is the relation between observation and the possession of an hypothesis?
9. Explain the nature of the development of an hypothesis. Why does such development require systematic knowledge?
10. Is a barren hypothesis the same as an unverifiable one?
11. Distinguish between 'proof' and 'verification.'
12. Why is analogy such an important source of hypotheses?
13. What is meant by 'false analogy'? Give at least five examples.
14. Take any two important theories generally accepted to-day and study their history. Exactly what was the problem which they were developed to explain?
15. What danger is there in arguing from the assumption that society is an organism?

CHAPTER XIX

Analyze the following examples of inductive argumentation, and state the method implied. Do the facts seem to you to be sufficient to found a rule upon?

1. It may be a coincidence merely; but, if so, it is remarkably strange that while the chloroform has not changed, while the constitutions of the patients have not changed, where the use of the inhaler is the rule, there are frequent deaths from chloroform; whilst in Scotland and Ireland, where the use of the inhaler is the exception, deaths are proportionally rare.[1]
2. Sir Charles Lyell, by studying the fact that the river Ganges yearly conveys to the ocean as much earth as would form sixty of the great pyramids of Egypt, was enabled to infer that the ordinary slow causes now in operation upon the earth would account for the immense geological changes that have occurred, without having recourse to the less reasonable theory of sudden catastrophes.
3. Take a bottle of charged water, slightly warmer than a given temperature registered by the thermopile, and mark the deflection it causes. Then cut the string which holds it and the cork will be driven out by the elastic force of the carbonic acid gas. The gas performs its works, and in so doing it consumes heat and the deflection of the thermopile shows that the bottle is cooler than before, heat having been lost in the process.
4. Any one who examines the records will soon find out for himself that those students who 'scatter' most in their choice of studies are those who accomplish least in any of them; and when he sees this he ought to realize the harm that can be done by a system of absolutely free electives.[2]
5. It was a general belief at St. Kilda that the arrival of a ship gave all the inhabitants colds. Dr. John Campbell took pains to ascertain the fact and to explain it as the effect of effluvia arising from human bodies; it was dis-

[1] Creighton. [2] Aikins.

covered, however, that the situation of St. Kilda renders a northeast wind indispensably necessary before a ship can make a landing. [1]

6. The great famine in Ireland began in 1845 and increased until it reached a climax in 1848. During this time agrarian crime increased very rapidly until, in 1848, it was more than three times as great as in 1845. After this it decreased with the return of better crops until, in 1851, it was only fifty per cent more than in 1845. It is evident from this that a close relation of cause and effect exists between famine and agrarian crime.
7. Wages in the United States are higher than in England, because the former country is a republic and has a protective tariff.
8. In Sir Humphry Davy's experiments upon the decomposition of water by galvanism, it was found that, besides the two components of water, oxygen and hydrogen, an acid and an alkali were developed at the two opposite poles of the machine. The insight of Davy conjectured that there might be some hidden cause of this portion of the effect: the glass containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it, so that the water would have no share in their production. . . . By the substitution of gold vessels for glass, without any change in the effect, he at once determined that the glass was not the cause. Employing distilled water, he found a marked diminution of the quantity of acid and alkali evolved; yet there was enough to show that the cause, whatever it was, was still in operation. . . . He now conceived that the perspiration from the hands touching the instruments might affect the case, as it would contain common salt, and an acid and an alkali would result from its decomposition under the agency of electricity. By carefully avoiding such contact, he reduced the quantity of the products still further until no more than slight traces of them were perceptible. What remained of the effect might be traceable to impurities of the atmosphere, de-

[1] Hibben.

composed by contact with the electrical apparatus. An experiment determined this: the machine was put under an exhausted receiver, and, when thus secured from atmospheric influence, it no longer evolved the acid and the alkali.[1]

9. In a simple fracture of the ribs if the lung be punctured by a fragment, the blood effused into the pleural cavity, although freely mixed with air, undergoes no decomposition. That is not the case if air enter directly through a wound in the chest. This difference in result must be causally connected with special circumstances — viz., passage of air through tissues in the lungs.[2]

10. "The two females were then seated upon two chairs placed near together, their heels resting on cushions, their lower limbs extended, with the toes elevated and the feet separated from each other. The object in this experiment was to secure a position in which the ligaments of the knee-joints should be made tense and no opportunity offered to make pressure with the foot. We were pretty well satisfied that the displacement of the bones requisite for the sounds could not be effected unless a fulcrum were obtained by resting one foot upon the other or on some resisting body. The company, seated in a semicircle, quietly waited for the 'manifestations' for more than half an hour. . . . On resuming the usual position on the sofa, the feet resting on the floor, knockings very soon began to be heard. . . . The conclusion seemed clear that the Rochester knockings emanate from the knee joint." [3]

11. Give two examples for each of the methods, preferably from ordinary experience rather than from science.

12. Show that Mill's Methods imply a large fund of relevant knowledge.

13. Analyze the causal relation in such a way as to bring out methods of testing supposed uniformities.

14. In what way is the Method of Difference an advance upon the Method of Agreement?

[1] Gore, *The Art of Scientific Discovery.*

[2] Russell.

[3] Description of the exposure of a spiritualistic séance, quoted by Podmore, and adapted by Taylor.

15. By what methods should the following problems be investigated?
    (1) An individual becoming sick after a meal.
    (2) The possibility of a connection between rag-weed and hay fever.
    (3) The effect of a new tariff upon the prosperity of a country.
    (4) Education and the decrease of superstition.
    (5) The relation between mosquitos and malaria.
    (6) Bad eyes and criminality.
    (7) A crime and the arrest of a suspected person.
    (8) The rapidity of growth of trees and the kind of soil.
    (9) Poverty and tuberculosis.
    (10) The effect of radium upon cancer.

CHAPTER XX

1. When are statistics resorted to? How do they enable us to grasp a field which would otherwise be unmanageable?
2. Why is definition of the problem and of the terms used so important?
3. Name the stages of any thorough statistical investigation. What are the special dangers confronting each of these stages?
4. State the law of statistical regularity, and try to explain its foundation.
5. In what sciences are statistics of greatest significance? Why?
6. Look up cases of erroneously interpreted statistics and show wherein the error lies.
7. "In Sweden the population and the smallpox mortality have both been known year by year since 1774. Before vaccination the mortality from smallpox for thirty years averaged 2045 per million. With permissive vaccination from 1802 to 1816 it was reduced to 480; during seventy-seven years of compulsory vaccination the mortality averaged 155 per million; and for ten years ending 1894 it has been down to 2 per million. . . .

"If we compare the rate of smallpox mortality in the different countries, we see an enormous difference between the well vaccinated and the badly vaccinated populations. Here is a table, given by Dr. Edwardes, of the mortality rates per million in the five years 1889 to 1893: —

	Smallpox mortality per million
Germany	2.3
England and Wales	13.6
Chief French towns	147.6
Italy	180.8
Belgium	253
Austria	313
Spain	638
Russia, three years only, including Asiatic Russia	836

"In Germany, vaccination and revaccination are both compulsory. In the other countries revaccination was, at that time at least, nowhere enforced." [1]

8. "More men than women die every year. This is due to the greater mortality attending the life of the male." [2] What does this mean? How can it be possible? "In Germany 109 men die each year for every 100 women." What can we conclude from this? Why is it better to know that in Germany 28.6 out of every thousand males die each year and 25.3 out of every thousand females? Since everybody must die sooner or later how is it possible that there should be in any country (or in the whole world) a permanently greater death-rate for one sex than for the other?
9. Show the importance of the qualitative side of social phenomena. Why are quantitative data apt to be misleading in certain fields?

[1] *Edinburgh Review*, vol. 189, pp. 350–52.
[2] Mayo-Smith, *Statistics and Sociology*.

10. Interpret the following statistics: —

A table giving the percentage of accidents occurring according to the time of the day

Time	*Germany*	*Italy*
12–3 A.M.	1.93	1.09
3–6 A.M.	2.55	2.47
6–9 A.M.	13.87	15.40
9–12 A.M.	28.42	29.20
12–3 P.M.	13.81	14.55
3–6 P.M.	26.62	26.48
6–9 P.M.	9.25	7.83
9–12 P.M.	3.85	2.54

11. Study carefully some one chapter of Mayo-Smith's *Statistics and Sociology* with especial attention to the reflective analysis which he gives at the end of the chapter.
12. What statistical investigations would seem to you very important for a modern, democratic society?
13. In the United States census of 1890, the question of color — whether black, mulatto, quadroon, or octoroon — was asked. Why was such a question faulty from the standpoint of statistics?
14. Point out the sociological bearing of the following statistics from the United States *Life Tables:* Of 100,000 negro females, 18,507 die before the age of one year, while of 100,000 white females, 10, 460 die before reaching the same age.
15. It is undeniable that tenancy is increasing in the United States. From the Twelfth Census, we have the following comparison: —

	Total number of farms	*Number operated by*		
		Owners	*Cash tenants*	*Share tenants*
1880	4,008,907	2,984,306	322,357	702,244
1900	5,739,657	3,713,371	752,920	1,273,366

The Marxian Socialist maintains that this increase proves concentration in farming. Does any other explanation suggest itself to you?

CHAPTER XXI

1. Under what conditions can science predict with accuracy the occurrence of particular events? Give concrete examples.
2. Are effects always found when their causes are present? Be prepared to defend your answer by means of instances.
3. What does each of the following propositions mean?
   (1) The child will probably catch the measles.
   (2) The earth will probably grow colder in the distant future.
   (3) The probability in favor of a long life for a man of his profession is very great.
4. Distinguish between 'probability' and 'objective chance.' How can you harmonize the existence of probability with the Principle of the Uniformity of Nature?
5. Is there any difference between saying that a certain horse is the most likely to win the race and saying that it is likely to win? [1]
6. How should life-insurance statistics be interpreted? Can they be made to apply to any one individual? If so, in what sense?
7. What reply can be made to the following? "You say that the prisoner is probably guilty. I grant it. But this only means that the prisoners in most cases of this sort are guilty. It does not mean that this particular prisoner has even a touch of guilt. Your very use of the word 'probable' is a confession that for all you know he may be absolutely innocent. How then can you ask the jury to condemn him to an awful fate?" [2]
8. Discuss the following statement: "Nine times out of ten we can act only on probability."
9. What is the probability that a die will fall with the same side up four times in succession?
10. What is the probability that you will draw either an ace or a five of hearts from a full deck of cards?

[1] Aikins. [2] Aikins.

11. "The regularities of the mass have no compelling force over the individual."[1] Against what fallacy is this statement directed?

CHAPTER XXII

1. Give some common examples of the use of an arithmetical average. Explain such an expression as the 'average rate of interest.'
2. Show that percentage of increase in the population of a town or city is a case of arithmetical average.
3. What average would be of most use for the following?
   (1) The standard size of a manufactured article.
   (2) A stock of hats for a clothing store.
   (3) The physical estimation of a varsity crew.
   (4) The salary of teachers.
4. Find, or work out, a set of statistics and discover the mode and the median.
5. Examine critically the phrase, 'the average man.' What danger of misinterpretation lurks behind the phrase?
6. Why are graphical methods being used so freely at the present time?
7. Find at least five devices for graphical representation in common use and compare their effectiveness.[2]
8. Look up cases of graphical representation in the popular magazines and make a report upon at least four which particularly attracted your attention.
9. "An average must not be a mere numerical average, for that amounts to nothing. It must be a *typical average*, expressive of about what the real condition of things is." Explain this statement in the light of the preceding chapters.

CHAPTER XXIII

1. Contrast 'scientific investigation' and 'judicial proof' as regards their methods and social surroundings.
2. Estimate the following arguments: —
   (1) If you believe in the survival of the fittest, you must believe that this old manuscript was one of

[1] Mayo-Smith. [2] See Brinton on this question.

the best of its time, for it is the only one that has survived.

(2) This book is authentic; then why should we not believe what it says?

(3) This text of Cicero dates from the twelfth century and that dates only from the fourteenth; then why is n't this a better one to go by than that?

(4) This text has great value for the historian, for it was restored at infinite pains, and there is now every reason to believe that it is substantially correct.

(5) "Come and have your fortune told by Blank's system of palmistry. No man of science has ever disputed the claims of this system." [1]

3. Take any striking criminal case before the courts and work out the circumstantial evidence for and against the issue in the way suggested in the text.
4. What is, in your opinion, the comparative probative value of circumstantial and testimonial evidence?
5. Connect the possibility of a plurality of causes with the methods used in explaining events.
6. What seems to you to be a reasonable doubt?
7. Examine the following case. Do you think that there was a reasonable doubt of Bradford's guilt?

"Jonathan Bradford, in 1736, kept an inn, in Oxfordshire, on the London road to Oxford. He bore a very exceptional character. Mr. Hayes, a gentleman of fortune, being on his way to Oxford, on a visit to a relation, put up at Bradford's. He there joined company with two gentlemen, with whom he supped, and, in conversation, unguardedly mentioned that he had then about him a sum of money. In due time they retired to their respective chambers; the gentlemen to a two-bedded room, leaving, as is customary with many, a candle burning in the chimney corner. Some hours after they were in bed, one of the gentlemen, being awake, thought he heard a groan in an adjoining chamber; and this being repeated, he softly awaked his friend. They listened together, and the groans increasing, as of one dying and in pain, they both

[1] Aikins.

instantly arose and proceeded silently to the door of the next chamber, whence they had heard the groans, and, the door being ajar, saw a light in the room. They entered, and perceived a person weltering in his blood in the bed, and a man standing over him with a dark lantern in one hand and a knife in the other! The man seemed as petrified as themselves, but his terror carried with it all the terror of guilt. The gentlemen soon discovered that the murdered person was the stranger with whom they had that night supped, and that the man standing over him was their host. They seized Bradford directly, disarmed him of his knife, and charged him with being the murderer. He assumed, by this time, the air of innocence, positively denied the crime, and asserted that he came there with the same humane intentions as themselves; for that, hearing a noise, which was succeeded by a groaning, he got out of bed, struck a light, armed himself with a knife for his defense, and was but that minute entered the room before them. These assertions were of little avail; he was kept in close custody till the morning, and then taken before a neighboring justice of the peace. Bradford still denied the murder, but, nevertheless, with such apparent indications of guilt, that the justice hesitated not to make use of this most extraordinary expression, on writing out his mittimus, 'Mr. Bradford, either you or myself committed this murder.' "

8. How does the mind work in passing judgment upon a very complex case? Study your own method of reaching a conclusion in some particular problem of a high degree of complexity.
9. Analyze one of Conan Doyle's Sherlock Holmes stories to see just how the circumstantial evidence is used both to lead and to mislead.
10. What is the fallacy of 'neglected factor'? Give at least two examples which have come within your experience.[1]

[1] If the teacher can secure the book, he will find many interesting examples for all the difficulties confronting evidence in Wigmore's *Principles of Judicial Proof.* These examples are too long to quote in the present text.

11. Look up the investigation of some historical myth like that of William Tell. What is its interest to the logician?
12. Newton was of the opinion that oral tradition could be trusted for eighty years after the event. Others have named forty years. Why are such estimations rather absurd?
13. It is asserted by many recent historians of the American Revolution that our school histories contain a deal of myth along with a fair percentage of fact. What causes might lead to legend and myth about the founders of the American Republic?

CHAPTER XXIV

1. What is the nature of explanation?
2. Take any two problems which have arisen lately in your experience and try to grasp the exact character of the explanation given them.
3. What is meant by the 'sentiment of rationality'? Show that it depends upon the constitution of our minds.
4. Is explanation possible apart from concepts?
5. Apply Dewey's analysis of the rôle of concepts to at least two cases of interpretation, stressing identification, supplementation, and placing in a system.
6. What difference of setting have the two terms, 'proof' and 'explanation'? What have they in common?
7. How are induction and deduction united in the process of explanation? Illustrate by examples either from practical life or from science.
8. Are all systems equally tentative? Can you think of any system which seems to you to be in final shape?
9. How would you distinguish between 'general' and 'specific' explanation?
10. What principle would you adopt as a means of arranging the sciences in an orderly fashion? Do you think that such an ordering of them gives a penetrative classification?
11. Why is mathematics usually spoken of as 'deductive'? Does this seem to you the best term that could be used?

12. Examine historical investigation in the light of your present knowledge of logic. Show especially that all facts are judgments.

CHAPTER XXV

1. Distinguish clearly between 'consistent thinking' and 'true thinking.'
2. What is meant by 'logical necessity'?
3. State and explain the 'Laws of Thought.'
4. What principles are implied in all investigation which seeks to determine truth?
5. How would you formulate your conception of the internal criteria of truth?
6. Explain the conception of degrees of belief. Relate your views to the modality of judgments.
7. Does language express adequately the different degrees of belief we actually hold?
8. Show that the meaning of knowledge is intimately bound up with the meaning of truth.
9. Read Tennyson's *In Memoriam*, XXXIV. Does it seem to you to imply an appeal to a standard of truth other than the logical?
10. Be prepared to defend James's position or Clifford's according as the one or the other strikes you as the sounder.

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(Kindly arranged by Mr. Douglas Clapperton)

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